[This question paper contains 6 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1154 **D**

Unique Paper Code : 2352201102

Name of the Paper : DSC: Elements of Discrete Mathematics

Name of the Course : B.A. (Prog.)

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. **All** questions are compulsory. Marks are indicated.

1. (a) Prove $(p \Rightarrow q) \equiv ((\sim p) \vee q)$. Further construct the truth table to determine whether the statement $(p \wedge q) \Rightarrow p$ is a tautology or not. (7.5)

P.T.O.

(b) Let $A = \mathbb{R} \times \mathbb{R}$. Define the following relation R on A :

(a, b) R (c, d) if and only if $a^2 + b^2 = c^2 + d^2$.

(i) Show that R is an equivalence relation on A.

(ii) Find all the equivalence classes of R.

(7.5)

(c) Let $f : \mathbb{R} \times \mathbb{R} \to \mathbb{R} \times \mathbb{R}$ be a function defined as

$f(x,y) = (2x - y,\ x - 2y)\ \forall\ (x,y) \in \mathbb{R} \times \mathbb{R}$.

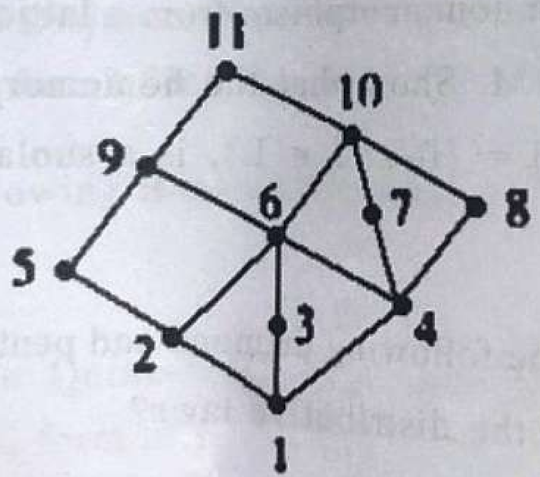
(i) Show that f is one to one.

(ii) Find f^{-1}. (7.5)

2. (a) Let A = {1,2,3,5,6,10,15,30}. Consider the partial order '≤' of divisibility on the set A defined as $a \leq b$ if and only if a divides b. Draw the Hasse diagram of the poset and determine if the poset is linearly ordered. (7.5)

(b) Let $A = \mathbb{Z}^+$ be the set of positive integers and B be the set of positive even integers. Consider $\leq$ to be the usual partial order of "less than or equal to" on sets A and B. Show that $(A, \leq)$ and $(B, \leq)$ are isomorphic posets. (7.5)

(c) Let $A = \{1,2,3,4,5,6,7,8,9,10,11\}$ be the poset whose Hasse diagram is shown below.



Let $B = \{6, 7, 10\}$. Find, if they exist,

(i) All maximal and minimal elements of the poset A.

(ii) All upper bounds and lower bounds of B.

(iii) The least upper bound and the greatest lower bound of B. (7.5)

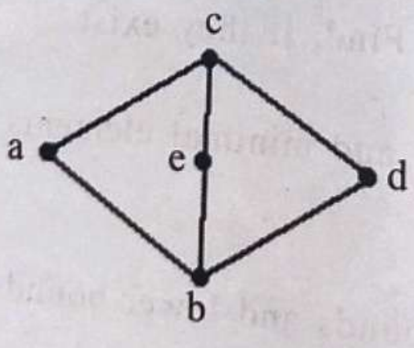
P.T.O.

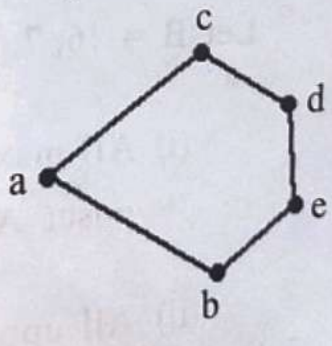
3. (a) Let L = P(A), where A = {a, b}. Draw the Hasse diagram for $(P(A), \cap, \cup)$ and find the complement of each element of P(A), where P(A) represents the power set of A. (7.5)

(b) Define a sublattice of a lattice L. Show that the interval $[x,y] = \{l \in L : x \leq l \leq y\}$, is a sublattice for any two elements $x, y \in L$ with $x \leq y$. (7.5)

(c) If f is a homomorphism from a lattice L to another lattice M. Show that the homomorphic image of L, $f(L) = \{f(l) : l \in L\}$, is a sublattice of M. (7.5)

4. (a) Does the following diamond and pentagonal lattices satisfy the distributive laws? (7.5)





6.

(b) Show that $(\mathbb{N}, lcm, gcd)$ is a distributive lattice. (7.5)

(c) Let L be a distributive lattice. Show that if $x \vee y = z \vee y$ and $x \wedge y = z \wedge y$ for every $x, y, z \in L$, then $x = z$. Also, prove that every element of L has at most one complement. (7.5)

5. (a) What is Karnaugh map? Use Karnaugh map diagram to find a minimal form of the function

$$wxyz + wxy\bar{z} + wx\bar{y}\bar{z} + w\bar{x}yz + w\bar{x}\bar{y}z + w\bar{x}\bar{y}\bar{z} + \bar{w}x\bar{y}z + \bar{w}\bar{x}yz + \bar{w}\bar{x}y\bar{z}$$ (7.5)

(b) What is Disjunctive normal form and Conjunctive normal form? Find the DN form and CN form of the following Boolean functions $\overline{(x\bar{y} + xz)} + \bar{x}$. (7.5)

(c) Use the Quine-McCluskey method to find the minimal form of

$$wx'y'z + w'xy'z' + wx'y'z' + w'xyz + w'x'y'z' + wxyz + wx'yz + w'xyz' + w'x'yz'.$$ (7.5)

6. (a) Let $f(x,y,z) = x\bar{z} + xyz + y\bar{z}$. Find the implicants, prime implicants and essential prime implicants. (7.5)

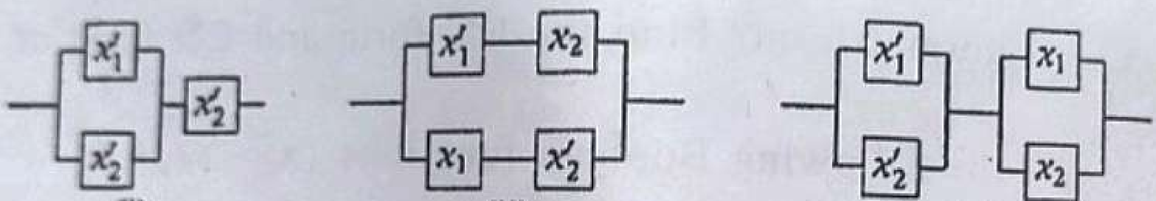
P.T.O.

(b) Construct a logic circuit by using inverters, AND gates and OR gates to produce these outputs

(i) $x\bar{y}z + xyz$

(ii) $xy\bar{z} + y\bar{z} + \bar{x}y$ (7.5)

(c) Determine which of the following contact diagrams give equivalent circuits:



(i) (ii) (iii)

(7.5)

[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 3174 **D**

Unique Paper Code : 2272201102

Name of the Paper : Basic Mathematics For Economic Analysis

Name of the Course : **BA (Prog) NEP-UGCF DSC-2**

Semester : I

Duration : 3 Hours Maximum Marks : 90

छात्रों के लिए निर्देश

1. इस प्रश्न-पत्र के मिलते ही ऊपर दिए गए निर्धारित स्थान पर अपना अनुक्रमांक लिखिए ।
2. प्रश्नपत्र को दो खंडों में बांटा गया है।
3. खंड अ से किन्हीं पाँच प्रश्नों और खंड ब से पाँच प्रश्नों के उत्तर दीजिए।
4. खण्ड अ के सभी प्रश्न छह अंकों के हैं।
5. खण्ड ब के सभी प्रश्न बारह अंकों के हैं।
6. साधारण कैलकुलेटर की अनुमति है।
7. इस प्रश्न-पत्र का उत्तर अंग्रेजी या हिंदी किसी एक भाषा में दीजिए, लेकिन सभी उत्तरों का माध्यम एक ही होना चाहिए ।

(खंड अ)

किन्हीं पांच प्रश्नों को हल करने का प्रयास कीजिए।

सभी प्रश्न छः अंकों के हैं।

1. निर्धारित करें कि क्या निम्नलिखित अनुक्रम अभिसरण या विचलन करते हैं:

P.T.O.

(i) $\left\{\frac{n^3-1}{n^2-2}\right\}$ (ii) $\left\{(-1)^{n-1}\frac{1}{2^n}\right\}$

2. निम्नलिखित सीमाएं ज्ञात कीजिये :

(i) $\lim_{x\to 1^-}\frac{-1}{\sqrt{1-x}}$ (ii) $\lim_{x\to 3}\frac{x^3-3x^2+9x-27}{x-3}$

3. मान लीजिए कि दिल्ली विश्वविद्यालय में छात्रों की कुल संख्या को सार्वभौमिक सेट U द्वारा निरूपित किया जाता है। आगे मान लीजिए कि E अर्थशास्त्र का अनुसरण करने वाले छात्रों की कुल संख्या को दर्शाता है। C वाणिज्य का अनुसरण करने वाले छात्रों की संख्या को दर्शाता है और S विज्ञान का अनुसरण करने वाले छात्रों की संख्या को दर्शाता है।

(a) यदि अर्थशास्त्र, वाणिज्य और विज्ञान के छात्रों के बीच कोई सामान्य छात्र नहीं हैं, तो वेन आरेख में तीन सेट E, C और S आरेख बनाइए।

(b) यदि अर्थशास्त्र के सभी छात्र वाणिज्य के छात्र हैं और कुछ वाणिज्य के छात्र विज्ञान के छात्र हैं लेकिन अर्थशास्त्र का कोई भी छात्र विज्ञान का अध्ययन नहीं कर रहा है, तो वेन आरेख कैसा दिखेगा?

4. दिया गया है - $A=\begin{pmatrix} a_{11} & 0 \\ 0 & a_{22} \end{pmatrix}$ and $x=\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$. दिखाइए कि $x'Ax$ वर्गों का भारित योग देता है।

5. 'मैट्रिक्स की रैंक' शब्द को परिभाषित कीजिए। क्या रैंक केवल वर्ग मैट्रिक्स के लिए परिभाषित है? आइडेन्टटी मैट्रिक्स की रैंक क्या है, I_3।

6. दिया गया है - $u=\begin{pmatrix}5\\1\end{pmatrix}$ और $v=\begin{pmatrix}0\\3\end{pmatrix}$, $2u+3v$ को ग्राफिक रूप से ज्ञात कीजिए।

7. दिया गया फलन है -

$$f(x) = 3x^2 - 12x + 13$$

$f'(x)=0$ पर x का मान निर्धारित कीजिये।

(खंड ब)

किन्हीं **पांच** *प्रश्नों को हल करने का प्रयास कीजिए।*

सभी प्रश्न बारह अंकों के हैं।

1. निम्नलिखित मैट्रिक्स के साथ गुणन के सहयोगी नियम का परीक्षण कीजिये :

$$A=\begin{pmatrix}5 & 3\\0 & 5\end{pmatrix}\quad B=\begin{pmatrix}-8 & 0 & 7\\1 & 3 & 2\end{pmatrix}\quad C=\begin{pmatrix}1 & 0\\0 & 3\\7 & 1\end{pmatrix}$$

2. मैट्रिक्स को उल्टा करके समीकरणों की निम्न प्रणाली को हल कीजिये।

$$2x - y = 2$$
$$3y + 2z = 16$$
$$3z + 5x = 21$$

3. अंतर्जात चर Q_d, Q_s और p में समीकरणों की निम्नलिखित प्रणाली पर विचार कीजिये :

$$Q_d = Q_s$$
$$Q_d = a - bp \quad (a, b > 0)$$
$$Q_s = -c + dp \quad (c, d > 0)$$

4. उपरोक्त मॉडल के लिए संतुलन समाधान को ज्ञात कीजिए और समाधान को तर्कसंगत होने के लिए आवश्यक शर्तें बताएं। समाधान को ग्राफिक रूप से दिखाइए।

$$h(x) = \frac{1+\left(\frac{1}{x}\right)}{\sqrt{(x-1)(x-2)}}$$

$$f(x) = \frac{\ln(x-1)}{x}$$

5. मान लीजिए कि ज्यामितीय श्रृंखला का तीसरा पद 12 है और श्रृंखला का छठा पद 144 है।

(a) श्रृंखला का पहला पद और सामान्य अनुपात ज्ञात कीजिए।

(b) श्रृंखला की शर्तों तक का योग ज्ञात कीजिये। यदि n अनंत की ओर जाता है तो योग का क्या होगा?

6. निम्नलिखित डेरिवेटिव को हल कीजिए और उपयोग किए गए नियम को बताइए :

(i) $(1+x)^{1/2}$ (ii) $(x+1)^2 / x$

(iii) जब $y = u^2$ और $u = 1 - x^3$ हों तो dy/dx ज्ञात कीजिये।

(3000)

[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1153 D

Unique Paper Code : 2352571101

Name of the Paper : DSC: Topics in Calculus

Name of the Course : B.A. / B.Sc. (Prog.) with Mathematics as Non-Major/ Minor

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **Two** parts from each question.
3. **All** questions carry equal marks.

1 (a) Prove $\lim_{x \to 0} x \sin\left(\frac{1}{x}\right) = 0$ by $\in - \delta$ definition of the limit. Does $\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$ exist? Justify.

P.T.O.

(b) Is the function $f(x)=\begin{cases}\frac{|x|}{x}, & x\neq 0\\ 1 & x=0\end{cases}$ continuous at $x = 0$? Justify. If discontinuous, mention the type of discontinuity.

(c) When is a function said to be differentiable at $x = a$? Examine for differentiability of

$$f(x)=\begin{cases}x\tan^{-1}\frac{1}{x}, & x\neq 0\\ 0 & x=0\end{cases} \text{ at } x = 0.$$

2. (a) If $y=\left(x+\sqrt{1+x^2}\right)^m$. Prove

$$(1 + x^2)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$$

Hence find $y_n(0)$.

(b) State Euler's theorem. Use it to prove that if

$$u=\cos^{-1}\frac{(x^2+y^2)}{(x+y)}, \text{ then } x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=-\cot u.$$

(c) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ show

(i) $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=\frac{3}{x+y+z}$

(ii) $\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$.

3. (a) State Taylor's theorem with Lagrange form of remainder. Expand log sin x in powers of $(x-2)$ using Taylor's theorem.

(b) State Lagrange's mean value theorem. Use it to prove that

$$\frac{x}{1+x} < \log(1+x) < x \ \forall \ x > 0.$$

(c) Find $\lim_{x\to 0} \frac{\tan^2 x - x^2}{x^2 \tan^2 x}$.

4. (a) State Cauchy's mean value theorem and verify it for $f(x) = x^2$, $g(x) = x^3$ in [1,2]. If $f(x) = \frac{1}{x^2}$, $g(x) = \frac{1}{x}$ in [a, b], then show that there exists $c \in (a, b)$ such that 'c' is the harmonic mean of a & b.

(b) Verify Rolle's theorem for

(i) $x^3 - 6x^2 + 11x - 6$ in [1,3]

(ii) $(x-a)^2(x-b)^3$, $x \in [a,b]$

P.T.O.

(c) If $\lim_{x \to 0} \frac{\sin 3x - a \sin x}{x^3}$ is finite find the value of 'a' and the limit.

5. (a) Find asymptotes of the following curve

$$y^3 - 5xy^2 + 8x^2y - 4x^3 - 3y^2 + 9xy - 6x^2 + 2y - 2x - 1 = 0$$

(b) Trace the curve $y^2(4 - x) = x^3$.

(c) Find the reduction formula for $\int \sin^m x \cos^n x \, dx$ and hence find

$$\int \sin^2 x \cos^3 x \, dx$$

6. (a) Define point of inflection of a curve. Find points of inflection for

(i) $y = -x^3 + x^2 - x + 1$

(ii) $y = e^{-x^2}$

(b) Trace the curve $r = 2 \sin 3\theta$.

(c) Find reduction formula for $\int \cos^n x \, dx$ and hence find $\int \cos^3 x \, dx$.

[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1152 **D**

Unique Paper Code : 2352571101

Name of the Paper : Topics in Calculus

Name of the Course : **BA / B.Sc. (Prog.) with Mathematics as Non-Major/ Minor**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. This question paper has **six** questions in all.
3. Attempt any **two** parts from each question.
4. **All** questions are compulsory.

Unit – I

1. (a) State $\varepsilon - \delta$ definition of a limit and use it to prove that $\lim_{x\to a} \cos x = \cos a$. (7.5)

P.T.O.

(b) Define continuity of a function and show that the function $f(x) = |x|$ is continuous at $x = 0$ but not derivable at $x = 0$. (7.5)

(c) Check whether the function $f(x)=\begin{cases} x\dfrac{e^{1/x}-1}{e^{1/x}+1}, & x \neq 0 \\ 0, & x=0 \end{cases}$.

is derivable at the origin or not. (7.5)

2. (a) If $y = \cos(m \sin^{-1} x)$, prove that

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 - m^2)y_n = 0.$$

(7.5)

(b) If $y = e^{-kt} \cos(pt + c)$, show that

$$\frac{d^2y}{dt^2} + 2k\frac{dy}{dt} + n^2y = 0 \text{ where } n^2 = p^2 + k^2.$$ (7.5)

(c) Define homogeneous function of degree n. If $f(x,y)$ is a homogeneous function of variables x and y of degree n, prove that

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf$$ (7.5)

Unit – II

3. (a) Give the geometrical interpretation of Rolle's theorem. Verify the Rolle's theorem for the function:

$f(x) = |x|$ in $[0,1]$. (7.5)

(b) State and prove Lagrange's mean value theorem. (7.5)

(c) State Cauchy's mean value theorem and calculate the values of c for which the following pair of functions

$f(x) = \sin x$ and $g(x) = \cos x$ in $[-\pi/2, 0]$

satisfy the condition of Cauchy's mean value theorem. (7.5)

4. (a) State Taylor's Theorem and hence expand $\log\sin(x + h)$ in terms of h. (7.5)

(b) Assuming $R_n(x) \to 0$ as $n \to \infty$, find the Maclaurin's series expansion of $e^x \sin x$. (7.5)

(c) Evaluate the following :

(i) $\lim_{x\to 0} \dfrac{\tan x - x}{x^2 \tan x}$ (4)

P.T.O.

(ii) $\lim_{x\to 0} \frac{x - \sin x}{x^3}$. (3.5)

Unit – III

5. (a) Find all the asymptotes of the curve

$$x^3 + 2x^2y - xy^2 - 2y^3 + 3xy + 3y^2 + x + 1 = 0.$$ (7.5)

(b) (i) Find the points of inflexion for the curve

$y^2 = (x-1)^2(x-2)$. (3.5)

(ii) Find the range of values of x in which the curve $y = x^4 - 4x^3 + 18x^2 + 1$ is concave upwards and concave downwards. (4)

(c) Trace the curve $y(x^2 + 4a^2) = 8a^3$. (7.5)

6. (a) Find the nature of the origin on the curve $x^4 + y^3 + 2x^2 + 3y^2 = 0$. (7.5)

(b) Find the reduction formula for $\int \sin^m x \cos^n x dx$ where m and n are positive integers. Hence evaluate $\int_0^{\pi/2} \sin^3 x \cos^4 x dx$. (7.5)

(c) Trace the curve $r^2 = a^2 \cos 2\theta$. (7.5)

(500)

[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 3173 **D**

Unique Paper Code : 2272201102

Name of the Paper : Basic Mathematics For Economic Analysis

Name of the Course : **BA (Prog) NEP-UGCF DSC-2**

Semester : I

Duration : 3 Hours Maximum Marks : 90

छात्रों के लिए निर्देश

1. इस प्रश्न-पत्र के मिलते ही ऊपर दिए गए निर्धारित स्थान पर अपना अनुक्रमांक लिखिए ।
2. प्रश्नपत्र को दो खंडों में बांटा गया है।
3. खंड अ से किन्हीं पाँच प्रश्नों और खंड ब से पाँच प्रश्नों के उत्तर दीजिए।
4. खण्ड अ के सभी प्रश्न छह अंकों के हैं।
5. खण्ड ब के सभी प्रश्न बारह अंकों के हैं।
6. साधारण कैलकुलेटर की अनुमति है।
7. इस प्रश्न-पत्र का उत्तर अंग्रेजी या हिंदी किसी एक भाषा में दीजिए, लेकिन सभी उत्तरों का माध्यम एक ही होना चाहिए ।

(खंड अ)

किन्हीं पांच प्रश्नों को हल करने का प्रयास कीजिए।

सभी प्रश्न छः अंकों के हैं।

1. बताइए कि निम्नलिखित कथन सत्य हैं या असत्य।

P.T.O.

(a) $(x - 1)(x + 2)^2 = 0 \Rightarrow x = 1$

(b) $|x - a| \leq 1 \Rightarrow a - 1 \leq x \leq a + 1$

(c) किसी अनुक्रम का Nवाँ पद 0 में परिवर्तित होता है ⇒ अनुक्रम अभिसारी है।

2. किन्हीं दो समुच्चयों A और B के बीच सममित अंतर को परिभाषित कीजिए।

$$A \Delta B = (A \cup B) \setminus (A \cap B)$$

(i) वेन आरेख के अंदर यूनीवर्स में दो सेट A और B दिये गये हैं, दोनों सेटों के बीच सममित अंतर दिखाइये।

(ii) भाग (i) में उत्तर का उपयोग करके, दिखाइये कि

$$A \Delta B = (A \setminus B) \cup (B \setminus A)$$

जहां किसी भी दो सेटों के बीच के अंतर को दर्शाता है।

3. निम्नलिखित फंक्शंस का डोमेन ज्ञात कीजिये :

$$f(x) = \frac{x^2 + 3x + 2}{(x+3)(x-2)}$$

4. पता लगाइंये $A^2 - 3A + 3I$ जहाँ $A = \begin{pmatrix} 4 & 5 \\ 0 & -2 \end{pmatrix}$.

5. त्रि-आयामी इयूक्लिडियन स्पेस में, निम्नलिखित बिंदुओं के बीच की दूरी कितनी है?

(i) (3,2,8) और (0,–1,5)

(ii) (9,0,4) और (2,0,–4)

6. निम्नलिखित द्वारा परिभाषित कार्यों के डेरिवेटिव ज्ञात कीजिए :

(i) $(1+x)^{1/2}$ (ii) $\frac{6x^3}{(2-x)}$

7. (i) योग अभिव्यक्ति का विस्तार कीजिये : $\sum_{i=0}^{3}(x+i)^2$

(ii) योग रूप में व्यक्त कीजिये :

$1 + 1/x + 1/x^2 + 1/x^3 + 1/x^4$

(खंड ब)

किन्हीं पांच प्रश्नों को हल करने का प्रयास कीजिए।

सभी प्रश्न बारह अंकों के हैं।

1. निम्नलिखित कार्यों के ग्राफ के लिए एसिम्प्टोटस ज्ञात कीजिए :

(i) $f(x)=\sqrt{\frac{2+3x}{x-1}}$ (ii) $h(x)=\frac{6x^3+5x^2-3}{(2-x)(3-x)}$

2. निम्नलिखित श्रृंखला के अभिसरण या विचलन का निर्धारण करें :

(a) $\sum_{n=1}^{\infty}\frac{n+2}{n-3}$ (b) $\sum_{n=1}^{\infty}(0.99999)^n$

(c) $\sum_{n=1}^{\infty}\frac{1}{n^{0.00001}}$

(d) $(1+r)^{-1}+(1+r)^{-2}+(1+r)^{-3}+\ldots\ldots\ldots\ldots\ldots,$
जहाँ r एक धनात्मक स्थिरांक है।

3 'मैट्रिक्स की रैंक' शब्द को परिभाषित कीजिए। क्या रैंक केवल वर्ग मैट्रिक्स के लिए परिभाषित की गई है?

x को इस प्रकार ज्ञात कीजिये कि $\begin{pmatrix} 1 & x & 1 \end{pmatrix}\begin{pmatrix} 1 & 3 & 2 \\ 0 & 5 & 1 \\ 0 & 3 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ x \end{pmatrix}=(0)$

4. मैट्रिक्स के व्युत्क्रम होने के लिए आवश्यक शर्त बताइए। निम्नलिखित मैट्रिक्स को पलट कर समीकरणों की निम्नलिखित प्रणाली को हल कीजिए :

$$10x - 2y + z = 8$$
$$6x + 3z = 7$$
$$2x + 3y - z = 6$$

5. तीन अंतर्जात चर Q_d, Q_s और p में समीकरणों की निम्नलिखित प्रणाली पर विचार कीजिए :

$Q_d = Q_s$
$Q_d = 3 - p^2$
$Q_s = 6P - 4$

द्विघात सूत्र का उपयोग करके उपरोक्त मॉडल के लिए संतुलन समाधान ज्ञात कीजिए। समाधान को ग्राफिक रूप से चित्रित कीजिए।

6. मान लीजिए कि $A=\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{pmatrix}$. सत्यापित करें कि $A'A$ सममित है।

दिया गया $B=\begin{pmatrix} 2 & 8 \\ 3 & 0 \\ 5 & 1 \end{pmatrix}$

क्या AB परिभाषित है? AB की गणना कीजिए। क्या आप BA की गणना कर सकते हैं? क्यों?

[This question paper contains 6 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 2155 **C**

Unique Paper Code	:	62354343
Name of the Paper	:	DSC – Analytical Geometry and Applied Algebra
Name of the Course	:	**B.A. (Prog.)**
Semester	:	III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. **All** questions are compulsory.

3. Attempt any **two** parts from each question.

1. (a) Find an equation of the parabola with vertex $(5, -3)$ axis parallel to y-axis and passes through $(9, 5)$. Also sketch the curve. (6)

P.T.O.

(b) Find the equation of hyperbola that has vertices at (2, 4) and (10, 4) and foci are 10 unit apart. Sketch the curve also. (6)

(c) Describe the graph of the curve $3(x + 3)^2 + 4(y + 2)^2 = 12$. Also find its centre and foci. (6)

2. (a) Find the centre, foci, vertices of the curve $x^2 + 5y^2 + 4x = 1$. (6)

(b) Rotate the co-ordinate axes to remove the xy term of the curve

$$3x^2 + 2\sqrt{3}\,xy + y^2 - 8x + 8\sqrt{3}y = 0 .$$ (6)

(c) Identify and sketch the curve $2x^2 - y^2 + 6y = 3$. (6)

3. (a) Find the angle between the vectors $\vec{a} = \hat{i} - 2\hat{j} + 9\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$. Also find the orthogonal projection of vector $\vec{v} = 6\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = \hat{i} - 2\hat{j} - 2\hat{k}$. (6.5)

(b) Find the direction cosines of the vector $\vec{v} = 2\hat{i} + 3\hat{j} - 6\hat{k}$, if it makes angles α, β and γ with x-axis, y-axis and z-axis, respectively. Then show that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ and Find $\vec{u} \cdot \vec{v}$, if $|\vec{u}| = 5$, $|\vec{v}| = 13$ and $|\vec{u} \times \vec{v}| = 25$. (6.5)

(c) Let $\vec{u} = \hat{i} - 3\hat{j} + 2\hat{k}$, $\vec{v} = \hat{i} + \hat{j}$, and $\vec{w} = 2\hat{i} + 2\hat{j} - 4\hat{k}$. Find the length of $3\vec{u} - 5\vec{v} + 2\vec{w}$. Also find the volume of the parallelopiped with adjacent edges $\vec{u}$, $\vec{v}$ and $\vec{w}$. (6.5)

4. (a) (i) Find the center and radius of the sphere: $x^2 + y^2 + z^2 - 2x + 8y - 4z = 4$. (3)

(ii) Sketch the graph of $x = z^2$ in 3-space. (3.5)

(b) (i) Find an equation of the plane passing through (1, −2, 2) that is perpendicular to the line $x - 5 = t$, $y + 3 = 3t$, $z = 2t$. (3.5)

(ii) Find the area of the triangle that is determined by the points

P(3, −1, 0), Q(−2, 0, 1) and R(0, 1, 3). (3)

(c) Find an equation of the sphere with centre (2, 1, −3) that is tangent to the plane

$$x - 3y + 2z - 4 = 0.$$ (6.5)

5. (a) Show that the lines :

L_1: $x = 1 + 4t$, $y = 5 - 4t$, $z = -1 + 5t$

L_2: $x = 2 + 8t$, $y = 4 - 3t$, $z = 5 + t$

are skew lines and find the distance between them. (6)

(b) (i) Find the distance between the point (1, −4, −3) and the plane

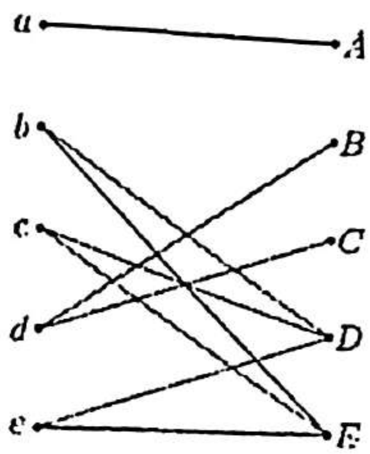
$$2x - 3y + 6z = -1.$$ (6)

(ii) Determine whether the line :

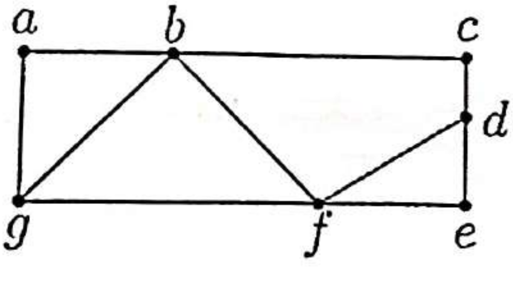
L: $x = 3 + 8t$, $y = 4 + 5t$, $z = -3 - t$ is parallel to the plane $x - 3y + 5z = 12$. (6)

(c) Find the volume of the tetrahedron with vertices P(1, 2, 0), Q(2, 1, 3), R(−1, 0, 1) and S(3, −2, 3). (6)

6\. (a) (i) Find a matching or explain why none exists for the following graph : (3)



(ii) Find all sets of two vertices whose removal disconnects the remaining graph. (3.5)



(b) Three pitchers of sizes 7 litres, 4 litres and 3 litres are given. Only 7 litres pitcher is full. Find a minimal sequence of pouring to make the quantity in three pitchers as 2 litres, 2 litres and 3 litres. (6.5)

(c) Construct a Latin square of order 5. Does the following latin square represent the multiplication table of a group of order 6? If not, give appropriate reason. (6.5)

A	B	C	D	E	F
B	A	F	E	C	D
C	F	B	A	D	E
D	C	E	B	F	A
E	D	A	F	B	C
F	E	D	C	A	B

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 2134 **C**

Unique Paper Code : 62354343

Name of the Paper : Analytic Geometry and Applied Algebra

Name of the Course : **B.A. (Prog.)**

Semester : III CBCS (LOCF)

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each questions
4. Each question carries **12.5** marks

1. (a) Identify and sketch the curve

$$(x + 2)^2 = -(y + 2)$$

And, also label the focus, vertex and directrix.

(b) Sketch the ellipse

$$(x + 3)^2 + 4(y - 5)^2 = 16$$

And, also label the foci, vertices and ends of major and minor axis.

(c) Describe the graph of the equation :

$$x^2 - y^2 - 4x + 8y - 21 = 0.$$

2. (a) Find an equation for the ellipse with length of minor axis 8 and with foci $(0, \pm 3)$ and also sketch it.

(b) Find an equation for the parabola that has its vertex at (1,2) and its focus at (4,2). Also, state the reflection property of parabola..

(c) Find the equation of the hyperbola with vertices $(0, \pm 8)$ and asymptotes $y = \pm\frac{4}{3}$.

3. (a) Consider the equation $x^2 - xy + y^2 - 6 = 0$. Rotate the coordinate axis to remove xy term and then identify the type of conic represented by the above equation.

(b) Identify and sketch the curve $xy = 1$.

(c) Let $x'y'$-coordinate system be obtained by rotating an xy- coordinate system through an angle $\alpha = 30$. Find the new $x'y'$ coordinate of the point whose an xy-coordinates are (2,4).

4. (a) Describe the surface whose equation is given:

$$x^2 + y^2 + z^2 + 10x + 4y + 2z - 19 = 0$$

(b) Using vector, find the area of the triangle that is determined by the points $P_1(2,2,0)$, $P_2(-1,0,2)$ and $P_3(0,4,3)$. Let $u = i - 3j + 2k$, $v = i + j$ and $w = 2i + 2j - 4k$. Find the volume of the parallelepiped with adjacent edges u, v and w.

(c) Find the orthogonal projection of $v = i + j + k$ on b.

5. (a) Find distance between the skew lines:

L_1: $x = 1 + 4t,\ y = 5 - 4t,\ z = -1 + 5t,\ -\infty < t < \infty$

L_2: $x = 2 + 8t,\ y = 4 - 3t,\ z = 5 + t,\ -\infty < t < \infty$

(b) Find the equation for the line L of intersection of the planes

$2x - 4y + 4z = 6$ and $6x + 2y - 3z = 4$

(c) Let a be the angle between the vectors $u = 2i + 3j - 6k$ and $v = 2i + 3j = 6k$.

(i) Use the dot product to find $\cos\alpha$.

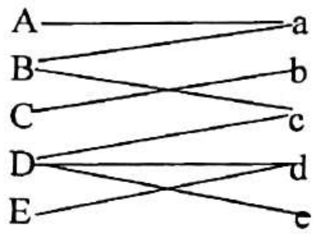
(ii) Use the cross product to find $\sin\alpha$.

(iii) Confirm that $\sin^2\alpha + \cos^2\alpha = 1$.

6. (a) Is the following a Latin square? Can it be a group with the multiplication operation defined?

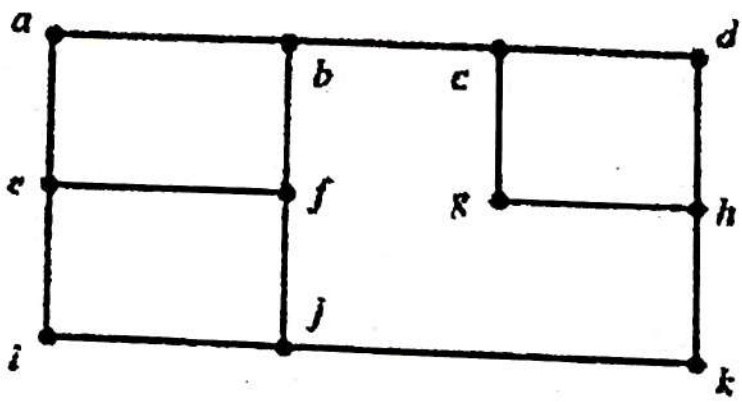
*	1	2	3	4	5
1	1	2	3	4	5
2	2	1	4	5	3
3	3	4	5	2	1
4	4	5	1	3	2
5	5	3	2	1	4

(b) Find a matching or explain why none exists for the following graph :



(c) The following figure represents a section of city's street map.

We want to position police at comers (vertices) so that they can keep every block (edge) under surveillance i.e. every edge should have a policeman atleast one of its comer. What is the smallest number of police that can do this job.



[This question paper contains 6 printed pages.]

Your Roll No..............

C

Sr. No. of Question Paper : 2142

Unique Paper Code : 62354343

Name of the Paper : Analytic Geometry and Applied Algebra

Name of the Course : **B.A. (Prog.)**

Semester : III CBCS (LOCF)

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each questions.
4. Each question carries **12.5** marks.

1. (a) Identify and sketch the curve :

$$y = 4x^2 + 8x + 5$$

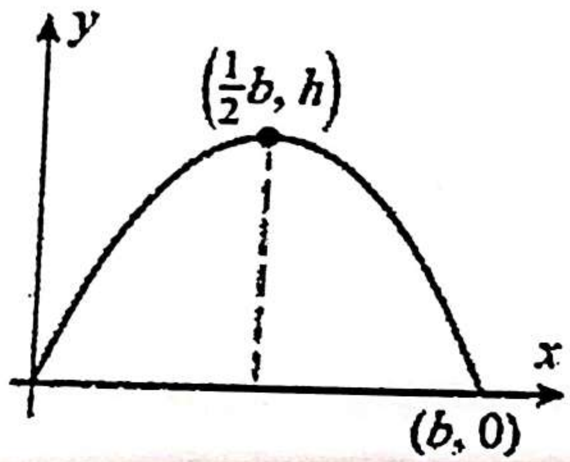
Also label the focus, vertex and directrix.

(b) Describe the graph of the curve:

$$x^2 + 9y^2 + 2x - 18y + 1 = 0$$

Find its foci, vertices and the ends of the minor axis.

(c) Find an equation for the parabolic arch with base b and height h, shown in the accompanying figure



2. (a) Find the equation for the parabola that has axis $y = 0$ and passes through (3, 2) and (2, –3).

(b) Find the equation for the ellipse that has foci (1,2) and (1,4) and minor axis of length 2.

(c) Describe the graph of the hyperbola :

$$x^2 - 4y^2 + 2x + 8y - 7 = 0$$

Also sketch its graph.

3. (a) If $\vec{a}, \vec{b}, \vec{c}$ are three mutually perpendicular unit vectors, then prove that

$$\|\vec{a} + \vec{b} + \vec{c}\| = \sqrt{3}$$

(b) Express $\vec{v}$ as the sum of a vector parallel to $\vec{b}$ and a vector orthogonal to $\vec{b}$ where

$$\vec{v} = 3\hat{i} + \hat{j} + 2\hat{k} \quad \text{and} \quad \vec{b} = 2\hat{i} + \hat{k}$$

(c) (i) Using vectors, find the area of triangle with vertices P(2, 2, 0), Q(1, 4, –5) and R(7, 2, 9).

(ii) Use scalar triple product to determine whether the vectors

$\vec{u} = \langle 5, -2, 1 \rangle, \vec{v} = \langle 4, -1, 2 \rangle$ and $\vec{w} = \langle 1, -1, 0 \rangle$ are co-planar.

4. (a) Consider the equation $x^2 - xy + y^2 + 12 = 0$. Rotate the coordinate axes to remove xy-terms. Then identify and sketch the curve.

(b) Let an x′y′-coordinate system be obtained by rotating an xy -coordinate system through an angle of $\theta = 45°$.

(i) Find the x′y′-coordinates of the point whose xy-coordinates are $\left(\sqrt{2}, \sqrt{2}\right)$.

(ii) Find an equation of the curve $x^2 + xy + 2y^2 + 6 = 0$ in x′y′-coordinates.

(c) Describe the surface whose equation is given as

$$x^2 + y^2 + z^2 + 2y - 6z + 5 = 0$$

5. (a) Find the distance from the point P(2, 5, –3) to the plane

$$\vec{r}.\left(6\hat{i} - 3\hat{j} + 2\hat{k}\right) = 4$$

(b) Find the equation of the plane through the points $P_1(2, 1, 4)$, $P_2(0, 0, -3)$ that is perpendicular to the plane $4x + y + 3z = 2$.

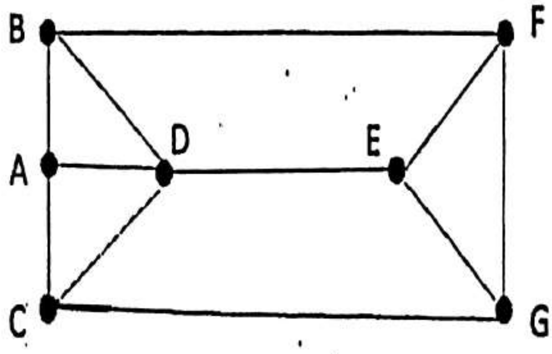
(c) Show that the lines L_1 and L_2 are parallel and find the distance between them

$L_1: x = 2 - t,\ y = 2t,\ z = 3 + t$

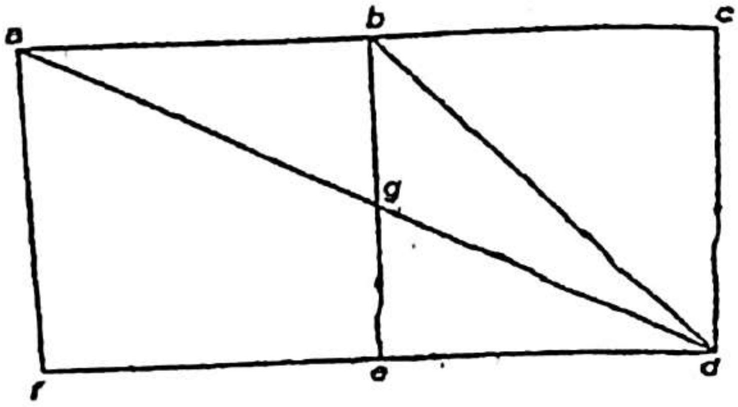
$L_2: x = -1 + 2t,\ y = 3 - 4t,\ z = 5 - 2t$

6. (a) Suppose a job placement agency wants to schedule interviews for candidates Ann, Judy and Carol with interviewers Al, Brian and Carl on Monday, Tuesday and Wednesday in such a way that each candidate gets interviewed by each interviewer. Solve this problem using a Latin Square.

(b) Find a vertex basis for the following graph:



(c) For the following graph, find a minimal edge cover and a maximal independent set of vertices.



[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1427 **C**

Unique Paper Code : 32351303

Name of the Paper : BMATH 307 – Multivariate Calculus

Name of the Course : **B.Sc. (H) Mathematics**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** sections are compulsory
3. Attempt any **Five** questions from each section. **All** questions carry equal marks

SECTION I

1. Let $f(x,y)=\dfrac{xy\left(x^2-y^2\right)x}{x^2+y^2}$ if $(x, y) \neq (0,0)$

$= 0$ otherwise

Show that $f(0, y) = -y$ and $f(x, 0) = x$ for all x and y.

2. Use incremental approximation to estimate the function $f(x, y) = \sin(xy)$ at the point

$$\left(\sqrt{\frac{\pi}{2}}+.01, \sqrt{\frac{\pi}{2}}-.01\right)$$

3. If $z = xy + f(x^2 + y^2)$, show that $y\, \partial z/\partial x - x\, \partial z/\partial y = y^2 - x^2$.

4. Assume that maximum directional derivative of f at $P_0(1,2)$ is equal to 50 and is attained in the direction towards $Q(3, -4)$. Find ∇f at $P_0(1, 2)$.

5. Find the absolute extrema of $f(x, y) = 2x^2 - y^2$ on the disk $x^2 + y^2 \leq 1$.

6. Use Lagrange multiplier to find the distance from (0, 0, 0) to plane $Ax + By + Cz = D$ where at least one of A, B, C is nonzero.

SECTION II

1. Compute the integral $\int_0^1 \int_x^{2x} e^{y-x} dy\, dx$ with the order of integration reversed.

2. Use Polar double integral to show that a sphere of radius α has volume $\frac{4}{3}\pi a^3$.

3. Compute the area of region D bounded above by line $y = x$, and below by circle $x^2 + y^2 - 2y = 0$.

4. Find the volume of the solid bounded above by paraboloid $z = 6 - x^2 - y^2$ and below by $z = 2x^2 + y^2$.

5. Evaluate $\iiint_D \frac{dx\,dy\,dz}{\sqrt{x^2+y^2+z^2}}$, where D is the solid sphere $x^2 + y^2 + z^2 \leq 3$.

6. Use a suitable change of variables to find the area of region R bounded by the hyperbolas $xy=1$ and $xy=4$ and the lines $y=x$ and $y=4x$.

SECTION III

1. Find the mass of a wire in the shape of curve C: $x = 3\sin t$, $y = 3\cos t$, $z = 2t$ for $0 \leq t \leq \pi$ and density at point (x, y, z) on the curve is $\delta(x, y, z) = x$.

2. Find the work done by force

$$\vec{F}(x,y,z) = \left(y^2 - z^2\right)\hat{i} + (2yz)\hat{j} - \left(x^2\right)\hat{k}$$

on an object moving along the curve C given by $x(t) = t$, $y(t) = t^2$, $z(t) = t^3$, $0 \leq t \leq 1$.

3. Use Green's theorem to find the work done by the force field

$$\vec{F}(x,y) = (3y - 4x)\hat{i} + (4x - y)\hat{j}$$

when an object moves once counterclockwise around the ellipse $4x^2 + y^2 = 4$.

4. Use Stoke's theorem to evaluate the surface integral

$$\iint_S \left(\text{curl } \vec{F}.N\right) dS$$

where $F = x\,i + y^2 j + z\,e^{xy}\,k$ and S is that part of surface $z = 1 - x^2 - 2y^2$ with $z \geq 0$.

5. Use divergence theorem to evaluate the integral $\iint_S \vec{F}.N\,dS$ where $\vec{F}(x,y,z) = (\cos yz)\hat{i} + e^{xz}\hat{j} + 3z^2\hat{k}$,

where S is hemisphere surface $z = \sqrt{4 - x^2 - y^2}$ together with the disk $x^2 + y^2 \leq 4$, in x-yplane.

6. Evaluate the line integral $\int_C \vec{F}.d\vec{R}$

Where $\vec{F}(x, y) = [(2x - x^2y)\,e^{-xy} + \tan^{-1} y]\,\hat{i} + \left[\frac{x}{y^2+1} - x^3 e^{-xy}\right]\hat{j}$ and C is the ellipse $9x^2 + 4y^2 = 36$.