DEC-2023

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 4386 **G**

Unique Paper Code : 32351502

Name of the Paper : Group Theory – II

Name of the Course : **B.Sc. (H) Mathematics**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Question No. **1** has been divided into **10** parts and each part is of **1.5** marks.
4. Each question from Q. Nos. **2** to **6** has **3** parts and each part is of **6** marks. Attempt any **two** parts from each question.

1. State true (T) or false (F). Justify your answer in brief.

   (i) Let G be a finite group of order 147 then it has a subgroup of order 49.

   (ii) There is a simple group of order 102.

P.T.O.

(iii) Dihedral Group D_{12} (having 24 elements) is isomorphic to the symmetric group S_3.

(iv) The action $z \cdot a = z + a$ of the additive group of integers Z on itself is faithful.

(v) The external direct product $G \oplus H$ is cyclic if and only if groups G and H are cyclic.

(vi) Trivial action is always faithful.

(vii) The group of order 27 is abelian.

(viii) The external direct product $Z_2 \oplus Z_6$ is cyclic.

(ix) Every Sylow p-subgroup of a finite group has order some power of p.

(x) A p-group is a group with property that it has atleast one element of order p.

2. (a) Prove that for every positive integer n, Aut $(Z_n) \cong U(n)$.

(b) Define Automorphism Aut(G) of a group G and Inner Automorphism Inn(G) of the group G induced by an element 'a' of G. Prove that $Aut(Z_5)$ is isomorphic to U(5), where $U(5) = \{1, 2, 3, 4\}$ is group under the multiplication modulo 5.

(c) Define characteristic subgroup of G. Prove that every subgroup of a cyclic group is characteristic.

3. (a) Prove that the order of an element of a direct product of finite number of finite groups is the least common multiple of the orders of the components in the elements. Find the largest possible order of an element in $Z_{30} \oplus Z_{20}$.

(b) Prove that if a group G is the internal direct product of finite number of subgroups H_1, H_2,.... H_n then G is isomorphic to $H_1 \oplus H_2 \oplus H_3 \ldots \oplus H_n$.

(c) Let G is an abelian group of order 120 and G has exactly three elements of order 2. Determine the isomorphism class of G.

4. (a) Show that the additive group R acts on x,y plane $R \times R$ by $r.(x,y)=(x+ry,y)$.

(b) Let G be a group acting on a non-empty set A. Define

(i) kernel of group action

(ii) Stabilizer of a in G, for $a \in A$

(iii) Prove that kernel is a normal subgroup of G.

(c) Define the permutation representation associated with action of a group on a set. Prove that the kernel of an action of group G on a set A is the same as the kernel of the corresponding permutation representation of the action.

P.T.O.

5. (a) Let G be a group acting on a non-empty set A. If $a, b \in A$ and $b = g \cdot a$ for some $g \in G$. Prove that $G_b = g\, G_a\, g^{-1}$ where G_a is stabilizer of a in G. Deduce that if G acts transitively on A then kernel of action is $\cap_{g \in G} g\, G_a\, g^{-1}$.

(b) Define the action of a group G on itself by conjugation. Prove it is a group action. Also find the kernel of this action.

(c) If G is a group of order pq, where p and q are primes, $p < q$, and p does not divide q-1, then prove that G is cyclic.

6. (a) State the Class Equation for a finite group G. Find all the conjugacy classes for quaternion group Q_8 and also, compute their sizes. Hence or otherwise, verify the class equation for Q_8.

(b) Use Sylow theorems to determine if a group of order 105 is not simple.

(c) State and prove Embedding theorem and use it to prove that a group of order 112 is not simple.

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[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 1553 **G**

Unique Paper Code : 2352011101

Name of the Paper : DSC-1 : Algebra

Name of the Course : **B.Sc. (H) Mathematics, UGCF-2022**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory and carry equal marks.
3. Attempt any **two** parts from each question.

1. (a) (i) Find a cubic equation with rational coefficients having the roots

$\frac{1}{2}, \frac{1}{2}+\sqrt{2}$, stating the result used.

(ii) Find an upper limit to the roots of

$x^5 + 4x^4 - 7x^2 - 40x + 1 = 0.$ (4+3.5)

P.T.O.

(b) Find all the integral roots of

$$x^4 + 4x^3 + 8x + 32 = 0.$$ (7.5)

(c) Find all the rational roots of

$$y^4 - \frac{40}{3}y^3 + \frac{130}{3}y^2 - 40y + 9 = 0.$$ (7.5)

2. (a) Express arg $(\bar{z})$ and arg $(-z)$ in terms of arg(z). Find the geometric image for the complex number z, such that $\arg(-z) \in \left(\frac{\pi}{6}, \frac{\pi}{3}\right)$. (2+2+3.5)

(b) Find $|z|$, arg z, Arg z, arg $\bar{z}$, arg$(-z)$ for $z = (1 - i)(6 + 6i)$ (7.5)

(c) Find the cube roots of $z = 1 + i$ and represent them geometrically to show that they lie on a circle of radius $(2)^{1/6}$. (7.5)

3. (a) Solve $y^3 - 15y - 126 = 0$ using Cardan's method. (7.5)

(b) Let n be a natural number. Given n consecutive integers, a, a + 1, a + 2, ..., a + (n–1), show that one of them is divisible by n. (7.5)

(c) Let a and b be two integers such that gcd(a, b) = g. Show that there exists integers m and n such that g = ma + nb. (7.5)

4. (a) Let a be an integer such that a is not divisible by 7. Show that $a \equiv 5^k \pmod 7$ for some integer k. (7.5)

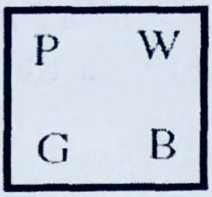
(b) Let a and b be two integers such that 3 divides $(a^2 + b^2)$. Show that 3 divides a and b both. (7.5)

(c) Solve the following pair of congruences, if possible. If no solution exists, explain why? (7.5)

$x + 5y \equiv 3 \pmod 9$

$4x + 5y \equiv 1 \pmod 9$

5. (a) Consider a square with four corners labelled as follows :



Describe the following motions graphically:

(i) R_0 = Rotation of 0 degree.

(ii) R_{90} = Rotation of 90 degrees counterclockwise.

(iii) R_{180} = Rotation of 180 degrees counterclockwise.

(iv) R_{270} = Rotation of 270 degrees counterclockwise.

(v) H = Flip about horizontal axis.

(vi) V = Flip about vertical axis.

(vii) D = Flip about the main diagonal.

(viii) D1 = Flip about the other diagonal.

Identify the motion that can act as identity under the composition of two motions. Further, find out the inverse of each motion. (3.5+1+3)

(b) Show that the set $G = (f_1, f_2, f_3, f_4\}$, is a group under the composition of functions defined as, $fog(x) = f(g(x))$ for f, g in G, where $f_1(x) = x$, $f_2(x) = -x$, $f_3(x) = 1/x$, $f_4(x) = -1/x$ for all non-zero real number x. (7.5)

(c) Define the inverse of an element in a group G. Show that $(a.b)^{-1} = b^{-1}.a^{-1}$ for all a, b in G. Further show that if $(a.b)^{-1} = a^{-1}.b^{-1}$ for all a, b in G, then G is Abelian. (4+3.5)

6. (a) Define Z(G), the center of a group G. Show that Z(G) is a subgroup of G. (2+5.5)

(b) Define order of an element a in group G. Further show that if order of a is n, and $a^m = e$, where m is an integer, then n divides m. (2+5.5)

(c) Find the generators of the cyclic group Z_{30}. Further describe all the subgroups of Z_{30} and find the generators of the subgroup of order 15 in Z_{30}. (2+3.5+2)

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[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 4469 **G**

Unique Paper Code : 32357501

Name of the Paper : DSE-I Numerical Analysis (LOCF)

Name of the Course : **B.Sc. (Hons.) Mathematics**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All **six** questions are compulsory.
3. Attempt any **two** parts from each question.
4. Use of non-programmable scientific calculator is allowed.

P.T.O.

1. (a) Discuss the order of convergence of the Newton Raphson method. (6)

(b) Perform three iterations of the Bisection method in the interval (1, 2) to obtain root of the equation $x^3 - x - 1 = 0$. (6)

(c) Perform three iterations of the Secant method to obtain a root of the equation $x^2 - 7 = 0$ with initial approximations $x_0 = 2$, $x_1 = 3$. (6)

2. (a) Perform three iterations of False Position method to find the root of the equation $x^3 - 2 = 0$ in the interval (1, 2). (6.5)

(b) Find a root of the equation $x^3 - 5x + 1 = 0$ correct up to three places of decimal by the Newton's

Raphson method with $x_0 = 0$. In how many iterations does the solution converge? Also write down the order of convergence of the method used. (6.5)

(c) Explain the secant method to approximate a zero of a function and construct an algorithm to implement this method. (6.5)

3. (a) Find an LU decomposition of the matrix

$$A = \begin{bmatrix} 2 & 7 & 5 \\ 6 & 20 & 10 \\ 4 & 3 & 0 \end{bmatrix}$$

and use it to solve the system $AX = [0 \quad 4 \quad 1]^T$. (6.5)

P.T.O.

(b) Set up the Gauss-Jacobi iteration scheme to solve the system of equations :

$$5x_1 + x_2 + 2x_3 = 10$$

$$-3x_1 + 9x_2 + 4x_3 = -14$$

$$x_1 + 2x_2 - 7x_3 = -33$$

Take the initial approximation as $X^{(0)} = (0,0,0)$ and do three iterations. (6.5)

(c) Set up the Gauss-Seidel iteration scheme to solve the system of equations :

$$6x_1 - 2x_2 + x_3 = 11$$

$$-2x_1 + 7x_2 + 2x_3 = 5$$

$$x_1 + 2x_2 - 5x_3 = -1$$

Take the initial approximation as $X^{(0)} = (1, 0, 0)$ and do three iterations. (6.5)

4. (a) Construct the Lagrange form of the interpolating polynomial from the following data :

x	0	1	3
f(x)	1	3	55

(6)

(b) Construct the divided difference table for the following data set and then write out the Newton form of the interpolating polynomial.

x	0	1	2	3
y	−1	0	15	80

Hence, estimate the value of f(1.5). (6)

(c) Obtain the piecewise linear interpolating polynomials for the function f(x) defined by the data :

x	–1	0	1	2
f(x)	3	–1	–3	1

(6)

5. (a) Derive second-order backward difference approximation to the first derivative of a function f given by

$$f'(x_0) \approx \frac{3f(x_0) - 4f(x_0 - h) + f(x_0 - 2h)}{2h}.$$ (6)

(b) Use the formula

$$f''(x_0) \approx \frac{f(x_0 + h) - 2f(x_0) + f(x_0 - h)}{h^2}$$

to approximate the second derivative of the function $f(x) = e^x$ at $x_0 = 0$, taking $h = 1, 0.1, 0.01$ and 0.001. What is the order of approximation.

(6)

(c) Approximate the derivative of $f(x) = 1 + x + x^3$ at $x_0 = 0$ using the first order forward difference formula taking h = ½, ¼ and 1/8 and then extrapolate from these values using Richardson extrapolation. (6)

6. (a) Using the trapezoidal rule, approximate the value of the integral $\int_3^7 \ln x \, dx$. Verify that the theoretical error bound holds. (6.5)

(b) Derive the Simpson's $1/3^{rd}$ rule to approximate the integral of a function. (6.5)

(c) Apply the modified Euler method to approximate the solution of the initial value problem

$\frac{dx}{dt} = 1 + \frac{x}{t}, 1 \le t \le 2, x(1) = 1$ taking the step size as $h = 0.5$.

(6.5)

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Your Roll No.....

Sr. No. of Question Paper : 4368 **G**

Unique Paper Code : 32351101

Name of the Paper : Calculus

Name of the Course : **B.Sc. (H) Mathematics (LOCF)**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** the sections are compulsory.
3. **All** questions carry equal marks.
4. Use of non-programmable scientific calculator is allowed.

SECTION – I

Attempt any **four** *questions from Section – I.*

1. Sketch the graph of the function $f(x) = x^4 - 4x^3 + 10$ by finding intervals where it increases and decreases, relative extrema, concavity and inflection points (if any).

P.T.O.

2. Evaluate the following limit

$$\lim_{x \to 0^+} \left(e^x - 1\right)^{1/\ln x}$$

3. Determine all the vertical and horizontal asymptotes to the curve

$$f(x) = \frac{x^2 - x - 2}{x - 3}.$$

4. A manufacturer estimates that when x units of a particular commodity are produced each month, the total cost (in dollars) will be

$$C(x) = \frac{1}{8}x^2 + 4x + 200$$

and all units can be sold at a price of $p(x) = 49 - x$ dollars per unit. Determine the price that correspond to the maximum profit.

5. If $y = e^{m \sin^{-1}x}$, Show that

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + m^2)y_n = 0.$$

SECTION – II

Attempt any **three** *questions from Section – II.*

6. Sketch the graph of the curve $r = 2 \cos 3\theta$ in polar coordinates.

7. Find an equation for a hyperbola that satisfies the condition that the curve has vertices $(0, \pm 3)$ and foci $(0, \pm 5)$.

8. Describe the graph of the equation :
$$9x^2 + 4y^2 - 18x + 24y + 9 = 0.$$

9. Identify and sketch the curve :
$$x^2 - xy + y^2 - 2 = 0.$$

SECTION – III

Attempt any **four** *questions from Section – III.*

10. Find the arc length of the parametric curve:
$x = (1 + t)^2$, $y = (1 + t)^3$ for $0 \leq t \leq 1$.

11. Find the area of the surface generated by revolving the curve $y = \sqrt{16 - x^2}$, $-2 \leq x \leq 2$ about the x-axis.

12. The region bounded by the curves $y = \sqrt{x}$, $x = 4$, $x = 9$ and $y = 0$ is revolved about the line $x = 0$. Compute the volume of the resulting solid.

13. Find the length of the catenary $y = 10 \cosh\left(\frac{x}{10}\right)$ from $x = -10$ to $x = 10$.

14. Evaluate $\int \sin^4 x \cos^2 x \, dx$.

P.T.O.

SECTION – IV

Attempt any **four** *questions from Section – IV.*

15. Find $\lim_{t\to 3}\left[t^2\hat{i}+\frac{\sin(2t-2)}{t-1}\hat{j}+e^{3t}\hat{k}\right]$.

16. If $R(t) = \ln(t^2 + 1)\,\hat{i} + (\tan^{-1} t)\,\hat{j} + \left(\sqrt{t^2+1}\right)\hat{k}$ is the position of a particle in space at time t. Find the angle between the velocity and acceleration vectors at time $t = 0$.

17. Determine all values of t for which the vector function $F(t) = (e^{2t} \sin 3t)\,\hat{i} + (t^2 \cos 3t)\,\hat{j}$ is continuous.

18. A golf ball is hit from the tee to a green with an initial speed of 125 ft/s at an angle of elevation of 45°. How long will it take for the ball to hit the green?

19. A shell is fired from ground level with muzzle speed of 750 ft/s at an angle of 25°. An enemy gun 20,000 ft away fires a shot 2 seconds later and the shells collide 50 ft above the ground at the same speed. What is the muzzle speed and angle of elevation of the second gun?

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Sr. No. of Question Paper	:	**1572**
Unique Paper Code	:	2352012302
Name of the Paper	:	DSC-8 : Riemann Integration
Programme	:	B.Sc. (Hons.) Mathematics (NEP-UGCF 2022)'
Semester	:	III
Duration : 3 Hours		Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory. Attempt any **Three** parts from each question.
3. All questions carry equal marks.

1. (a) Let $f: [-1,1] \to \mathbb{R}$ be defined as follows:

$$f(x) = \begin{cases} 2, & if\ x \in \mathbb{Q} \\ 3, & if\ x \notin \mathbb{Q} \end{cases}$$

Show that f is not integrable on [-1,1].

(b) Let $f: [a,b] \to \mathbb{R}$ be a bounded function. Show that if f is integrable on $[a,b]$, then for each $\varepsilon > 0$, there exists a $\delta > 0$ such that $U(f,P) - L(f,P) < \varepsilon$ for every partition P of $[a,b]$ with $\text{mesh}(P) < \delta$.

(c) Let $f(x) = 3x + 2$ over the interval [1,3]. Let P be a partition of [1,3] given by $P = \{1, 3/2, 2, 3\}$. Compute $L(f,P)$, $U(f,P)$ and $U(f,P) - L(f,P)$.

(d) Let $f: [a,b] \to \mathbb{R}$ be a bounded function. Show that if P and Q are any partitions of $[a,b]$, then $L(f,P) \leq U(f,Q)$. Hence show that $L(f) \leq U(f)$.

(e)

2. (a) Prove that a bounded function f is integrable on $[a,b]$ if and only if there exists a sequence of partitions $(P_n)_{n\in\mathbb{N}}$ of $[a,b]$, satisfying $\lim[U(f,P_n) - L(f,P_n)] = 0$.

(b) Suppose that a function f defined on $[a,b]$ is integrable on $[a,c]$ and $[c,b]$, where $c \in (a,b)$. Prove that f is integrable on $[a,b]$ and that $\int_a^b f = \int_a^c f + \int_c^b f$.

(c) Let $f: [a,b] \to \mathbb{R}$ be a bounded function. Show that if f is Riemann integrable on $[a,b]$, then it is (Darboux) integrable on $[a,b]$, and that the values of the integrals agree.

(d) For $t \in [0,1]$, let $F(t) = \begin{cases} 0 & \text{for } t < 1/3 \\ 1 & \text{for } t \geq 1/3. \end{cases}$

Let $f(x) = x^2$, where $x \in [0,1]$. Show that f is F-integrable and that $\int_0^1 f\, dF = f(1/3)$.

3. (a) Prove that every continuous function on $[a,b]$ is integrable on $[a,b]$.

(b) State and prove the Intermediate Value Theorem for Integrals.

(c) (i) Show that $\left|\int_{-2\pi}^{2\pi} x^2 cos^8(e^x)\, dx\right| \leq \frac{16\pi^3}{3}$.

(ii) Give an example of a function f on $[0, 1]$ that is not integrable for which $|f|$ is integrable on $[0, 1]$.

(d) Suppose that f and g are continuous functions on $[a, b]$ such that $\int_a^b f\, dx = \int_a^b g\, dx$. Prove that there exists x in $[a, b]$ such that $f(x) = g(x)$.

4. (a) If f and g are two integrable functions on $[a, b]$, then prove that $(f + g)$ is also integrable on $[a, b]$.

(b) Prove that every piecewise monotonically increasing function on $[a, b]$ is integrable on $[a, b]$.

(c) State Fundamental Theorem of Calculus-II. Hence or otherwise evaluate $\lim_{h \to 0} \frac{1}{h} \int_4^{4+h} e^{t^2}\, dt$.

(d) Let f be defined on $\mathbb{R}$ as

$$f(t) = \begin{cases} t & for\ t < 0 \\ t^2 + 1 & for\ 0 \leq t \leq 2 \\ 0 & for\ t > 2. \end{cases}$$

Determine the function $F(x) = \int_0^x f(t)\, dt$.

(i) At what points F is continuous?

(ii) At what points F is differentiable? Calculate F' at the points of differentiability.

5. (a) Find the volume of the solid generated when the region under the curve $y = x^2$ over the interval $[0, 2]$ is rotated about the line $y = -1$.

(b) Use cylindrical shells to find the volume of the solid generated when the region enclosed between $y = \sqrt{x}$, $x = 4$, $x = 9$ and the x-axis is revolved about the y-axis.

(c) Find the exact arc length of the curve $y = x^{\frac{2}{3}}$ from x = 1 to x = 8.

(d) The circle $x^2 + y^2 = r^2$ is rotated about the x-axis to obtain a sphere. Find the surface area of the sphere.

6. (a) Discuss the convergence of following improper integrals:

(i) $\int_0^1 \frac{1}{x \ln x}\, dx$ (ii) $\int_1^\infty \frac{dx}{\sqrt{x^3 + x}}$.

(b) Find a function f such that $\int_1^\infty f$ converges, but $\int_1^\infty \sqrt{f}$ does not converge. Justify your answer.

(c) Show that the improper integral $\int_0^1 t^{p-1}(1-t)^{q-1}\, dt$ converges if and only if p and q are positive.

(d) Show that

(i) $\Gamma(p+1) = p\,\Gamma(P)$ for all $p > 0$,

(ii) $\Gamma(n) = (n-1)!$ for all $n \in \mathbf{N}$.

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Sr. No. of Question Paper	:	**1572**
Unique Paper Code	:	2352012302
Name of the Paper	:	DSC-8 : Riemann Integration
Programme	:	B.Sc. (Hons.) Mathematics (NEP-UGCF 2022)'
Semester	:	III
Duration : 3 Hours		Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All questions are compulsory. Attempt any **Three** parts from each question.
3. All questions carry equal marks.

1. (a) Let $f: [-1,1] \to \mathbb{R}$ be defined as follows:

$$f(x) = \begin{cases} 2, & if\ x \in \mathbb{Q} \\ 3, & if\ x \notin \mathbb{Q} \end{cases}$$

Show that f is not integrable on [-1,1].

(b) Let $f: [a,b] \to \mathbb{R}$ be a bounded function. Show that if f is integrable on $[a,b]$, then for each $\varepsilon > 0$, there exists a $\delta > 0$ such that $U(f,P) - L(f,P) < \varepsilon$ for every partition P of $[a,b]$ with $\text{mesh}(P) < \delta$.

(c) Let $f(x) = 3x + 2$ over the interval [1,3]. Let P be a partition of [1,3] given by $P = \{1, 3/2, 2, 3\}$. Compute $L(f,P)$, $U(f,P)$ and $U(f,P) - L(f,P)$.

(d) Let $f: [a,b] \to \mathbb{R}$ be a bounded function. Show that if P and Q are any partitions of $[a,b]$, then $L(f,P) \leq U(f,Q)$. Hence show that $L(f) \leq U(f)$.

(e)

2. (a) Prove that a bounded function f is integrable on $[a,b]$ if and only if there exists a sequence of partitions $(P_n)_{n\in\mathbb{N}}$ of $[a,b]$, satisfying $\lim[U(f,P_n) - L(f,P_n)] = 0$.

(b) Suppose that a function f defined on $[a,b]$ is integrable on $[a,c]$ and $[c,b]$, where $c \in (a,b)$. Prove that f is integrable on $[a,b]$ and that $\int_a^b f = \int_a^c f + \int_c^b f$.

(c) Let $f: [a,b] \to \mathbb{R}$ be a bounded function. Show that if f is Riemann integrable on $[a,b]$, then it is (Darboux) integrable on $[a,b]$, and that the values of the integrals agree.

(d) For $t \in [0,1]$, let $F(t) = \begin{cases} 0 & \text{for } t < 1/3 \\ 1 & \text{for } t \geq 1/3. \end{cases}$

Let $f(x) = x^2$, where $x \in [0,1]$. Show that f is F-integrable and that $\int_0^1 f\, dF = f(1/3)$.

3. (a) Prove that every continuous function on $[a,b]$ is integrable on $[a,b]$.

(b) State and prove the Intermediate Value Theorem for Integrals.

(c) (i) Show that $\left|\int_{-2\pi}^{2\pi} x^2 cos^8(e^x)\, dx\right| \leq \frac{16\pi^3}{3}$.

(ii) Give an example of a function f on $[0, 1]$ that is not integrable for which $|f|$ is integrable on $[0, 1]$.

(d) Suppose that f and g are continuous functions on $[a, b]$ such that $\int_a^b f\, dx = \int_a^b g\, dx$. Prove that there exists x in $[a, b]$ such that $f(x) = g(x)$.

4. (a) If f and g are two integrable functions on $[a, b]$, then prove that $(f + g)$ is also integrable on $[a, b]$.

(b) Prove that every piecewise monotonically increasing function on $[a, b]$ is integrable on $[a, b]$.

(c) State Fundamental Theorem of Calculus-II. Hence or otherwise evaluate $\lim_{h \to 0} \frac{1}{h} \int_4^{4+h} e^{t^2}\, dt$.

(d) Let f be defined on $\mathbb{R}$ as

$$f(t) = \begin{cases} t & for\ t < 0 \\ t^2 + 1 & for\ 0 \leq t \leq 2 \\ 0 & for\ t > 2. \end{cases}$$

Determine the function $F(x) = \int_0^x f(t)\, dt$.

(i) At what points F is continuous?

(ii) At what points F is differentiable? Calculate F' at the points of differentiability.

5. (a) Find the volume of the solid generated when the region under the curve $y = x^2$ over the interval $[0, 2]$ is rotated about the line $y = -1$.

(b) Use cylindrical shells to find the volume of the solid generated when the region enclosed between $y = \sqrt{x}$, $x = 4$, $x = 9$ and the x-axis is revolved about the y-axis.

(c) Find the exact arc length of the curve $y = x^{\frac{2}{3}}$ from x = 1 to x = 8.

(d) The circle $x^2 + y^2 = r^2$ is rotated about the x-axis to obtain a sphere. Find the surface area of the sphere.

6. (a) Discuss the convergence of following improper integrals:

(i) $\int_0^1 \frac{1}{x \ln x}\, dx$ (ii) $\int_1^\infty \frac{dx}{\sqrt{x^3 + x}}$.

(b) Find a function f such that $\int_1^\infty f$ converges, but $\int_1^\infty \sqrt{f}$ does not converge. Justify your answer.

(c) Show that the improper integral $\int_0^1 t^{p-1}(1-t)^{q-1}\, dt$ converges if and only if p and q are positive.

(d) Show that

(i) $\Gamma(p+1) = p\,\Gamma(P)$ for all $p > 0$,

(ii) $\Gamma(n) = (n-1)!$ for all $n \in \mathbf{N}$.

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[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 1610 **G**

Unique Paper Code : 2352012303

Name of the Paper : Discrete Mathematics

Name of the Course : **B.Sc. (H) – DSC**

Semester : III

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** question by selecting **two** parts from each question.
3. Parts of the questions to be attempted together.
4. **All** questions carry equal marks.
5. Use of Calculator not allowed.

P.T.O.

1. (a) (i) Define covering relation in an ordered set and finite ordered set. Prove that if X is any set, then the ordered set $\wp(X)$ equipped with the set inclusion relation given by $A \leq B$ iff $A \subseteq B$ for all $A, B \in \wp(X)$, a subset B of X covers a subset A of X iff $B = A \cup \{b\}$ for some $b \in X - A$.

(ii) State Zorn's Lemma.

(b) (i) Give an example of an ordered set (with diagram) with more than one maximal element but no greatest element. Specify maximal elements also.

(ii) Define when two sets have the same cardinality. Show that

- $\mathbb{N}$ and $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$
- $\mathbb{Z}$ and $2\mathbb{Z}$

have the same cardinality.

(c) Let $\mathbb{N}_0$ be the set of whole numbers equipped with the partial order $\leq$ defined by $m \leq n$ if and only if m divides n. Draw Hasse diagram for the subset $S = \{1,2,4,5,6,12,20,30,60\}$ of $(\mathbb{N}_0, \leq)$. Find elements $a, b, c, d \in S$ such that $a \vee b$ and $c \wedge d$ does not exist in S.

2. (a) Define an order preserving map. In which of the following cases is the map $\varphi : P \to Q$ order preserving?

(i) $P = Q = (\mathbb{N}_0, \leq)$ and $\varphi(x) = nx$ ($n \in \mathbb{N}_0$ is fixed).

(ii) $P = Q = (\wp(\mathbb{N}), \subseteq)$ and φ defined by

$$\varphi(U) = \begin{cases} \{1\}, & 1 \in U \\ \{2\}, & 2 \in U \text{ and } 1 \notin U, \\ \emptyset, & \text{otherwise} \end{cases}$$

P.T.O.

where $\mathbb{N}_0$ be the set of whole numbers equipped with the partial order $\leq$ defined by $m \leq n$ iff m divides n and $\wp(\mathbb{N})$ be the power set of $\mathbb{N}$ equipped with the partial order given by $A \leq B$ iff $A \subseteq B$ for all $A, B \in \wp(\mathbb{N})$.

(b) For disjoint ordered sets P and Q define order relation on $P \cup Q$. Draw the diagram of ordered sets (i) 2×2 (ii) $3 \cup \overline{3}$ (iii) $M_2 \oplus M_3$ where $M_n = 1 \oplus \overline{n} \oplus 1$.

(c) Let $X = \{1,2,\ldots, n\}$ and define φ: $\wp(X) \to 2^n$ by $\varphi(A) = (\varepsilon_1,\ldots, \varepsilon_n)$ where

$$\varepsilon_i = \begin{cases} 1 & \text{if } i \in A \\ 0 & \text{if } i \notin A \end{cases}$$

Show that φ is an order-isomorphism.

3. (a) Let L and K be lattices and f: $L \to K$ a lattice homomorphism.

(i) Show that if $M \in \text{Sub } L$, then $f(M) \in \text{Sub } K$.

(ii) Show that if $N \in \text{Sub } K$, then $f^{-1}(N) \in \text{Sub}_0 L$, where $\text{Sub}_0 L = \text{Sub } L \cup \emptyset$.

(b) Let L be a lattice.

(i) Assume that $b \leq a \leq b \vee c$ for $a, b, c \in L$. Show that $a \vee c = b \vee c$.

(ii) Show that the operations $\vee$ and $\wedge$ are isotone in L, i.e. $b \leq c \Rightarrow a \wedge b \leq a \wedge c$ and $a \vee b \leq a \vee c$.

(c) Let L and M be lattices. Show that the product $L \times M$ is a lattice under the operations $\vee$ and $\wedge$ defined as

$$(x_1, y_1) \vee (x_2, y_2) := (x_1 \vee x_2,\ y_1 \vee y_2),$$

$$(x_1, y_1) \wedge (x_2, y_2) := (x_1 \wedge x_2,\ y_1 \wedge y_2)$$

P.T.O.

4. (a) Let L be a distributive lattice. Show that $\forall x, y, z \in$ L, the following laws are equivalent :

(i) $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$

(ii) $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$

(b) Define modular lattices. Show that every distributive lattice is modular. Is the converse true? Give arguments in support of your answer.

(c) (i) Prove that for any two elements x, y in lattice L, the interval

$[x,y] := \{a \in L | x \leq a \leq y\}$ is a sublattice of L.

(ii) Let f be a monomorphism from a lattice into a lattice M. Show that L is isomorphic to a sublattice of M.

5. (a) (i) Prove that $(x \wedge y)' = x' \vee y'$ and $(x \vee y)' = x' \wedge y'$ for all x, y in a Boolean algebra.

Deduce that $x \leq y \Leftrightarrow x' \geq y'$ for all $x, y \in B$.

(ii) Show that the lattice B = ({1,2,3,6,9,18}, gcd, lcm) of all positive divisors of 18 does not form a Boolean algebra.

(b) Find the conjunctive normal form of

$$(x_1 + x_2 + x_3)(x_1x_2 + x_1'x_3)'$$

(c) Use a Karnaugh Diagram to simplify

$$p = x_1x_2x_3 + x_2x_3x_4 + x_1'x_2x_4' + x_1' x_2x_3x_4' + x_1' x_2 x_4'$$

6. (a) Use the Quine-McCluskey method to find the minimal form of

$wxyz' + wxy'z' + wx'yz + wx'yz' + w'x'yz + w'x'yz' + w'x'y'z$

P.T.O.

(b) Draw the contact diagram and determine the symbolic representation of the circuit given by

$$p = x_1x_2(x_3+x_4) + x_1x_3(x_5+x_6)$$

(c) Give mathematical models for the following random experiments

(i) when in tossing a die, all outcomes and all combinations are of interest.

(ii) when tossing a die, we are only interested whether the points are less than 3 or greater than or equal to 3.

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[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 1591 **G**

Unique Paper Code : 2352011102

Name of the Paper : DSC-2 : Elementary Real Analysis

Name of the Course : **B.Sc. (H) Mathematics (UGCF-2022)**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **three** parts from each question.
3. **All** questions carry equal marks.

1. (a) Let $a \geq 0$, $b \geq 0$ prove that $a^2 \leq b^2 \Leftrightarrow a \leq b$.

P.T.O.

(b) Determine and sketch the set of pairs (x, y) on $\mathbb{R} \times \mathbb{R}$ satisfying the inequality $|x| \leq |y|$.

(c) Find the supremum and infimum, if they exist, of the following sets :

(i) $\left\{\sin\frac{n\pi}{2} : n \in \mathbb{N}\right\}$

(ii) $\left\{\left(\frac{1}{x} : x > 0\right)\right\}$

(d) Show that $\text{Sup}\left\{1+\frac{1}{n} : n \in \mathbb{N}\right\} = 2$.

2. (a) Let S be a non-empty bounded subset of $\mathbb{R}$. Let $a > 0$ and let $aS = \{as: s \in S\}$. Prove that

$$\text{Sup } (aS) = a(\text{ Sup } S)$$

(b) If x and y are positive rational numbers with $x < y$, then show that there exists a rational number r such that $x < r < y$.

(c) Show that $\inf\left\{\frac{1}{n} : n \in \mathbb{N}\right\} = 0$.

(d) Show that every convergent sequence is bounded. Is the converse true? Justify.

3. (a) Using definition of limit, show that

$$\lim_{n\to\infty} \frac{n^2+3n+5}{2n^2+5n+7} = \frac{1}{2}$$

(b) Show that if $c > 0$, $\lim_{n\to\infty} (c)^{1/n} = 1$.

(c) Show that, if $x_n \geq 0$ for all n, and $\langle x_n \rangle$ is convergent then $\left\langle \sqrt{x_n} \right\rangle$ is also convergent and

$$\lim_{n\to\infty} \sqrt{x_n} = \sqrt{\lim_{n\to\infty} x_n}$$

(d) Show that every increasing sequence which is bounded above is convergent.

4. (a) Let $x_1 = 1$ and $x_{n+1} = \sqrt{2x_n}$ for all n. Prove that $\langle x_n \rangle$ is convergent and find its limit.

(b) Prove that every Cauchy sequence is convergent.

(c) Show that the sequence $\langle x_n \rangle$ defined by

$$x_n = 1 + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{n!}, \text{ for all } n \in \mathbb{N}$$

is convergent.

(d) Find the limit superior and limit inferior of the following sequences :

(i) $x_n = (-1)^n \left(1 + \frac{1}{n}\right)$, for all $n \in \mathbb{N}$

(ii) $x_n = \left(1 + \frac{1}{n}\right)^{n+1}$, for all $n \in \mathbb{N}$

5. (a) Show that if a series Σa_n converges, then the sequence $\langle a_n \rangle$ converges to 0.

(b) Determine, if the following series converges, using the definition of convergence, $\sum \log\left(\frac{a_n}{a_{n+1}}\right)$ given that $a_n > 0$ for each n, $\lim_{n\to\infty} a_n = a$, $a > 0$.

(c) Find the rational number which is the sum of the series represented by the repeating decimal $0.\overline{987}$.

(d) Check the convergence of the following series :

(i) $\sum \frac{1}{2^n + n}$

(ii) $\sum \sin\left(\frac{1}{n^2}\right)$

6. (a) State the Root Test (limit form) for positive series. Using this test or otherwise, check the convergence of the following series

(i) $\sum\left(n^{1/n}-1\right)^n$

(ii) $\sum\left(\frac{n^{n^2}}{(n+1)^{n^2}}\right)$

(b) Check the convergence of the following series :

(i) $\sum_{n=2}^{\infty}\left(\frac{1}{n \log n}\right)$

(ii) $\sum\left(\frac{n!}{n^n}\right)$

(c) Define absolute convergence of a series. Show that every absolutely convergent series is convergent. Is the converse true? Justify your answer.

P.T.O.

(d) Check the following series for absolute or conditional convergence :

(i) $\sum(-1)^{n+1}\left(\frac{n}{n(n+3)}\right)$

(ii) $\sum(-1)^{n+1}\left(\frac{1}{n+1}\right)$

[This question paper contains 24 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 568 **G**

Unique Paper Code : 2272102302

Name of the Paper : Intermediate Macroeconomics I : Foundations of Aggregate Income Determination

Name of the Course : **B.A. (Hons) Economics**

Semester : III

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. This question paper is divided into **three** sections.
3. Use of simple calculator is allowed.
4. Answers maybe written either in English or Hindi; but the same medium should be used throughout the paper.

P.T.O.

छात्रों के लिए निर्देश

1. इस प्रश्न-पत्र के मिलते ही ऊपर दिए गए निर्धारित स्थान पर अपना अनुक्रमांक लिखिए ।
2. यह प्रश्न पत्र **तीन** खंडों में विभाजित है ।
3. साधारण कैलकुलेटर के प्रयोग की अनुमति है ।
4. इस प्रश्न-पत्र का उत्तर अंग्रेजी या हिंदी किसी एक भाषा में दीजिए, लेकिन सभी उत्तरों का माध्यम एक ही होना चाहिए ।

SECTION A (खंड क)

Question **1** *is compulsory.* (15×2=30)

प्रश्न **1** *अनिवार्य है ।*

1. (i) Assume that the economy is closed (i.e. no import or export). There is a decrease in taxes. What happens to interest rate in Medium Run due to the Labour Market?

(a) Increase

(b) Decrease

(c) Unchanged

(d) Indeterminate

(ii) The permanent income of a consumer in Friedman's model of consumption is

(a) Present value of her income/interest rate

(b) Present value of her income X interest rate

(c) Interest rate/ Present value of her income

(d) None of the above

(iii) Which of the following event increases natural rate of unemployment?

(a) Decrease in Government Spending

(b) Increase in Real Money Supply

(c) Decrease in competition amongst firms

(d) Weakening of Labor Union

(iv) Assume that expected inflation follows $\pi_t = \pi_{t-1}$. What inflation level is consistent with the government maintaining output below the natural level?

(a) Positive Inflation

(b) Negative Inflation

(c) Zero Inflation

(d) All of the above

(v) Which of the following variables will change in the long run in response to a demand shock if prices are able to adjust fully?

(a) Unemployment

(b) Output and unemployment

(c) Price level

(d) None of the above

(vi) The life cycle hypothesis implies that in a growing economy with increasing social security benefits for old-age people may

(a) Increase gross private savings

(b) Reduce gross private savings

(c) Increase gross government savings

(d) Increase aggregate savings

(vii) What happens to the short-run Aggregate Supply curve as the money wage rate rises?

(a) the short-run aggregate supply curve shifts rightward.

(b) the short-run aggregate supply curve shifts leftward.

(c) the long-run aggregate supply curve shifts rightward.

(d) both the long-run aggregate supply curve and the short-run aggregate supply curve shift leftward.

(viii) According to the pipeline theory, inventory investment is

(a) Countercyclical

(b) Procyclical

(c) Both (a) and (b)

(d) Constant overtime

(ix) Which of the following best explains how an economy could simultaneously experience high inflation and high unemployment?

(a) The government increases spending without increasing taxes.

(b) Inflationary expectations decline.

(c) Women and teenagers stay out of the labor force.

(d) Negative supply shocks cause factor prices to increase.

(x) According to Friedman the covariance between current consumption and transitory income is

(a) Greater than zero

(b) Less than zero

(c) Equal to zero

(d) Equal to 1

(xi) The modified Phillips curve tells us that the only way to reduce inflation is through

(a) unemployment rates higher than the natural rate

(b) expansionary fiscal policy

(c) unemployment rates lower than the natural rate

(d) contractionary fiscal policy

(xii) The natural rate of unemployment depends on all of the following except :

(a) The level of unemployment insurance

(b) The mark-up

(c) The bargaining power of workers with firms

(d) Money supply

(xiii) In terms of the Phillips Curve, wage indexation results in

(a) a weaker/flatter relationship between unemployment and changes in inflation.

(b) no effect on the relationship between unemployment and changes in inflation.

(c) a stronger/steeper relationship between unemployment and changes in inflation.

(d) an ambiguous effect that depends on the mark-up of firms.

(xiv) The stagnation thesis formed around 1940 was based on the ground that

(a) If the marginal propensity to consume (MPC) < average propensity to consume (APC) is accepted, then government expenditure share to GDP (g/y) must be increasing with GDP (y) to balance the drop in APC to maintain full-employment demand.

(b) If MPC < APC is accepted, then g/y must be decreasing with y to balance the increase in APC to maintain full employment demand.

(c) If MPC > APC is accepted, then g/y must be increasing with y to balance the drop in APC to maintain full employment demand.

(d) If MPC = APC is accepted, then g/y must be decreasing with y to balance the increase in APC to maintain full-employment demand.

(xv) According to the Phillips curve, unemployment will return to the natural rate when :

(a) Nominal wages are equal to expected wages

(b) Real wages are back at long-run equilibrium level

(c) Nominal wages are growing faster than inflation

(d) Inflation is higher than the growth of nominal wages

(i) मान लीजिए कि अर्थव्यवस्था बंद है (यानी कोई आयात या निर्यात नहीं होता है)। करों में कमी आई है। श्रम बाजार के कारण मध्यम अवधि के लिए ब्याज दर में क्या परिवर्तन होगा?

(क) वृद्धि

(ख) कमी

(ग) अपरिवर्तित

(घ) अनिश्चित

(ii) फ्रीडमैन के उपभोग मॉडल में उपभोक्ता की स्थायी आय क्या होती है

(क) उसकी आय/ब्याज दर का वर्तमान मूल्य

(ख) उसकी आय का वर्तमान मूल्य X ब्याज दर

(ग) ब्याज दर/उसकी आय का वर्तमान मूल्य

(घ) उपरोक्त में से कोई नहीं

(iii) निम्नलिखित में से कौन-सी घटना बेरोजगारी की प्राकृतिक दर को बढ़ाती है?

(क) सरकारी खर्च में कमी

(ख) वास्तविक धन की आपूर्ति में वृद्धि

(ग) फर्मों के बीच प्रतिस्पर्धा में कमी

(घ) श्रमिक संघ का कमजोर होना

(iv) मान लीजिए कि अपेक्षित मुद्रास्फीति $\pi_t = \pi_{t-1}$ का अनुसरण करती है। सरकार द्वारा उत्पादन को प्राकृतिक स्तर से नीचे बनाए रखने के लिए मुद्रास्फीति का स्तर क्या होगा?

(क) सकारात्मक मुद्रास्फीति

(ख) नकारात्मक मुद्रास्फीति

(ग) शून्य मुद्रास्फीति

(घ) उपरोक्त सभी

(v) यदि कीमतें पूरी तरह से समायोजित होने में सक्षम हैं, तो मांग के झटके की प्रतिक्रिया में निम्नलिखित में से कौन-सा चर लंबे समय में बदल जाएगा?

(क) बेरोजगारी

(ख) उत्पादन और बेरोजगारी

(ग) मूल्य स्तर

(घ) उपरोक्त में से कोई नहीं

(vi) जीवन चक्र परिकल्पना का तात्पर्य है कि आगे बढ़ती हुई अर्थव्यवस्था में वृद्ध लोगों के लिए सामाजिक सुरक्षा के लाभ में वृद्धि होगी

(क) सकल निजी बचत में वृद्धि

(ख) सकल निजी बचत में कमी

(ग) सकल सरकारी बचत में वृद्धि

(घ) कुल बचत में वृद्धि

(vii) मुद्रा मजदूरी दर बढ़ने पर अल्पकालिक सकल आपूर्ति वक्र में क्या होता है?

(क) अल्पकालिक समग्र आपूर्ति वक्र दाईं ओर स्थानांतरित होता है।

(ख) अल्पकालिक समग्र आपूर्ति वक्र बाई ओर स्थानांतरित होता है।

(ग) दीर्घकालिक समग्र आपूर्ति वक्र दाई ओर बदलता है।

(घ) दीर्घकालिक समग्र आपूर्ति वक्र और अल्पकालिक समग्र आपूर्ति वक्र दोनों बाई ओर स्थानांतरित होते हैं।

(viii) पाइपलाइन सिद्धांत के अनुसार, इन्वेंट्री निवेश क्या है?

(क) प्रतिचक्रीय

(ख) प्रोसाइक्लिकल

(ग) (क) और (ख) दोनों

(घ) समय के साथ स्थिर

(ix) निम्नलिखित में से कौन-सा सबसे अच्छी तरह से स्पष्ट करता है कि किसी अर्थव्यवस्था में एक साथ ही उच्च मुद्रास्फीति और उच्च बेरोजगारी कैसे हो सकती हैं?

(क) सरकार करों में वृद्धि किए बिना खर्च बढ़ाती है।

(ख) मुद्रास्फीति की अपेक्षाओं में कमी।

(ग) महिलाएं और किशोर श्रम शक्ति से बाहर रहते हैं।

(घ) नकारात्मक आपूर्ति झटके फलन कीमतों में वृद्धि का कारण बनते हैं।

(x) फ्रीडमैन के अनुसार वर्तमान खपत और अस्थायी आय के बीच सहसंयोजक क्या है?

(क) शून्य से अधिक

(ख) शून्य से कम

(ग) शून्य के बराबर

(घ) 1 के बराबर

(xi) संशोधित फिलिप्स वक्र हमें बताता है कि मुद्रास्फीति को कम करने का एकमात्र तरीका है

(क) बेरोजगारी दर प्राकृतिक दर से अधिक

(ख) विस्तारवादी राजकोषीय नीति

(ग) बेरोजगारी दर प्राकृतिक दर से कम

(घ) संकुचनकारी राजकोषीय नीति

(xii) बेरोजगारी की आधार दर निम्नलिखित सभी पर निर्भर करती है सिवाय इसके कि :

(क) बेरोजगारी बीमा का स्तर

(ख) मूल्य वृद्धि

(ग) फर्मों के साथ श्रमिकों की सौदेबाजी की शक्ति

(घ) धन की आपूर्ति

(xiii) फिलिप्स वक्र के संदर्भ में, मजदूरी सूचकांक का परिणाम क्या है

(क) बेरोजगारी और मुद्रास्फीति में परिवर्तन के बीच एक कमजोर/एकसमान संबंध।

(ख) बेरोजगारी और मुद्रास्फीति में परिवर्तन के बीच संबंधों पर कोई प्रभाव नहीं।

(ग) बेरोजगारी और मुद्रास्फीति में परिवर्तन के बीच एक मजबूत/तीव्र संबंध।

(घ) एक अस्पष्ट प्रभाव जो फर्मों की मूल्य वृद्धि पर निर्भर करता है।

(xiv) 1940 के आसपास गठित गतिरोध थीसिस इस आधार पर आधारित थी कि

(क) यदि सीमांत उपभोग प्रवृत्ति (MPC) < औसत उपभोग प्रवृत्ति (APC) को स्वीकार किया जाता है, तो APC में गिरावट को संतुलित करने के लिए पूर्ण रोजगार मांग को बनाए रखने हेतु सकल घरेलू उत्पाद में सरकारी व्यय का GDP (g/y) हिस्सा सकल घरेलू उत्पाद GDP (y) के साथ बढ़ना चाहिए।

(ख) यदि MPC < APC को स्वीकार कर लिया जाता है, तो पूर्ण रोजगार मांग को बनाए रखने के लिए APC में वृद्धि को संतुलित करने के लिए g/y को y के साथ घटाया जाना चाहिए।

(ग) यदि MPC > APC को स्वीकार कर लिया जाता है, तो पूर्ण रोजगार मांग को बनाए रखने के लिए APC में गिरावट को संतुलित करने के लिए g/y में वृद्धि होनी चाहिए।

(घ) यदि MPC = APC को स्वीकार कर लिया जाता है, तो पूर्ण रोजगार मांग को बनाए रखने के लिए APC में वृद्धि को संतुलित करने के लिए g/y को y के साथ घटाया जाना चाहिए।

(xv) फिलिप्स वक्र के अनुसार, बेरोजगारी की आधार दर पर वापस आ जाएगी जब :

(क) असमायोजित मजदूरी अपेक्षित मजदूरी के बराबर है

(ख) वास्तविक मजदूरी दीर्घकालिक संतुलन स्तर पर वापस आ गई है।

(ग) असमायोजित मजदूरी मुद्रास्फीति की तुलना में तेजी से बढ़ रही है

(घ) मुद्रास्फीति असमायोजित मजदूरी की वृद्धि से अधिक है

SECTION B (खंड ख)

Attempt any **6** *out of the following* **8** *questions.*
Each question carries **5** *marks.* (6×5=30)

निम्नलिखित 8 प्रश्नों में से किन्हीं 6 प्रश्नों का उत्तर दीजिए।
प्रत्येक प्रश्न के 5 अंक हैं।

2. (a) Describe the concept of NAIRU. What are its determinants?

(b) Can NAIRU change with time and across countries? (2+3)

(क) एनएआईआरयू की अवधारणा का वर्णन कीजिए। इसके निर्धारक तत्व क्या हैं?

(ख) क्या एनएआईआरयू समय और देशों के साथ बदल सकता है?

3. (a) The Phillips curve is $\pi_t = \pi_t^e + (\mu + z) - \alpha u_t$. Rewrite this relation as a relation between the deviation of the unemployment rate from the natural rate, inflation, and expected inflation.

(b) Online job websites are getting popular amongst job-seekers and enable them to connect with employers easily. What impact should this have on the natural rate of unemployment in an economy? (3+2)

(क) फिलिप्स वक्र $\pi_t = \pi_t^e + (\mu + z) - \alpha u_t$ इस संबंध को आधार दर, मुद्रास्फीति और अपेक्षित मुद्रास्फीति से बेरोजगारी दर के विचलन के बीच संबंध के रूप में फिर से लिखिए।

(ख) ऑनलाइन नौकरी वेबसाइटें नौकरी चाहने वालों के बीच लोकप्रिय हो रही हैं और उन्हें नियोक्ताओं के साथ आसानी से जुड़ने में सक्षम बनाती हैं। अर्थव्यवस्था में बेरोजगारी की आधार दर पर इसका क्या प्रभाव होना चाहिए?

4. Determine the price-earnings ratio for a stock using the arbitrage argument for financial investment. Can this measure be used to detect bubbles in the stock market? (5)

वित्तीय निवेश के लिए आर्बिट्रेज तर्क का उपयोग करके स्टॉक के लिए मूल्य-आय अनुपात निर्धारित कीजिए। क्या इस उपाय का उपयोग शेयर बाजार में उफान का पता लगाने के लिए किया जा सकता है?

5. "Disinflation typically leads to a period of higher unemployment." Explain. What is the basis of Lucas' assertion that the unemployment costs of disinflation can be reduced significantly? Do you expect faster disinflations to be associated with lower sacrifice ratios? (5)

"अवस्फीति आम तौर पर उच्च बेरोजगारी की अवधि की ओर ले जाती है।" स्पष्ट कीजिए। लुकास के दावे का आधार क्या है कि अवस्फीति की बेरोजगारी लागत को काफी कम किया जा सकता है? क्या आप उम्मीद करते हैं कि तीव्र अवस्फीति कम त्याग अनुपात से जुड़ी होगी?

6. (a) Compute the real interest rate using the exact formula and the approximation formula for $i = 4\%$; $\pi^e = 2\%$.

(b) Explain possible reasons for the firms to pay higher than market-clearing wages. (2+3)

(क) सटीक सूत्र और अनुमान सूत्र का उपयोग करके आधार ब्याज दर की गणना कीजिये $i = 4\%$; $\pi^e = 2\%$.

(ख) फर्मों को बाजार-समाशोधन मजदूरी से अधिक भुगतान करने के संभावित कारणों को स्पष्ट कीजिए।

7. Following the intertemporal optimising model of consumption, derive the intertemporal consumption relation with the interest rate (r) and future-utility discounting factor (δ). Provide a graph along with the economic reasoning behind the typical consumption path to be obtained for different relative magnitudes of r and δ. (3+2)

खपत के इंटरटेम्पोरल ऑप्टिमाइजिंग मॉडल का पालन करते हुए, ब्याज दर (r) और भविष्य-उपयोगिता छूट कारक (δ) के साथ इंटरटेम्पोरल खपत संबंध प्राप्त कीजिए। r और δ के विभिन्न सापेक्ष परिमाणों के लिए प्राप्त किए जाने वाले विशिष्ट उपभोग पथ के पीछे आर्थिक तर्क के साथ एक रेखाचित्र तैयार कीजिए।

8. Discuss an alternative theory of consumption that modifies the intertemporal budget constraint used in the intertemporal optimising models of consumption. Flow does this alternative theory modify the consumption path proposed in Ando-Modigliani's model of consumption (i.e., *Life Cycle Hypothesis*)? (5)

उपभोग के एक वैकल्पिक सिद्धांत पर चर्चा कीजिए जो खपत के इंटरटेम्पोरल ऑप्टिमाइजिंग मॉडल में उपयोग किए जाने वाले इंटरटेम्पोरल बजट बाधा को संशोधित करता है। यह वैकल्पिक सिद्धांत एंडो-मोदिग्लिआनी के उपभोग के मॉडल (यानी, जीवन चक्र परिकल्पना) में प्रस्तावित उपभोग पथ को कैसे संशोधित करता है?

9. Fiscal policy as well as Monetary policy cannot change the level of output in the medium run. Why is then monetary policy considered neutral but not fiscal policy? (5)

राजकोषीय नीति के साथ-साथ मौद्रिक नीति मध्यम अवधि में उत्पादन के स्तर को नहीं बदल सकती है। तो फिर मौद्रिक नीति को तटस्थ क्यों माना जाता है लेकिन राजकोषीय नीति को नहीं?

SECTION C (खंड ग)

Attempt any **3** *out of the following* **4** *questions. Each question carries* **10** *marks.* (3×10=30)

निम्नलिखित **4** प्रश्नों में से किन्हीं **3** प्रश्नों का उत्तर दीजिए। प्रत्येक प्रश्न के लिए **10** अंक हैं।

10. In a faraway country, the total population is 5000 people (all of them non-institutional civilian people), 3000 are working and 250 are looking for a job.

(a) What is the size of the labour force? What are the participation rate and unemployment rate? (3)

(b) In this economy, the labour productivity is 1. And the wage-setting process is described by

$\frac{W}{P^e} = Z - 100u$, Where Z is the unemployment insurance provided by the government and u is the unemployment rate. What is the Price Setting equation for a mark-up level $\mu = 1.5$? Show graphically and mathematically. (2+1)

(c) What is the natural unemployment rate (show it graphically as well), natural level of employment and natural level of output if $Z = 5$? (2+1+1)

एक दूर देश में, कुल आबादी 5000 लोग हैं (उनमें से सभी गैर-संस्थागत नागरिक हैं), 3000 काम कर रहे हैं और 250 नौकरी की तलाश में हैं।

(क) श्रम शक्ति का आकार क्या है? भागीदारी दर और बेरोजगारी दर क्या है?

(ख) इस अर्थव्यवस्था में, श्रम उत्पादकता 1 है। और मजदूरी-निर्धारण प्रक्रिया का वर्णन किया गया है। $\frac{W}{P^e} = Z - 100u$, जहां Z सरकार द्वारा प्रदान किया जाने वाला बेरोजगारी बीमा है और न बेरोजगारी दर है। मूल्य वृद्धि स्तर $\mu = 1.5$ के लिए मूल्य निर्धारण समीकरण क्या है? रेखांकन और गणितीय रूप से दर्शाएं।

(ग) आधार बेरोजगारी दर क्या है (इसे रेखाचित्र के रूप से भी दर्शाएंइस, रोजगार का मूल स्तर और उत्पादन का मूल स्तर यदि $Z = 5$ है?

11. How does Lucas's supply relation describe the possibility of the output in any economy to deviate from its full capacity level? What characteristic(s) of the rational expectation is (are) are required to prove the policy effective proposition. Using the model proposed by Sargent and Wallace (1976), explain the possibility of ineffectiveness of expansionary monetary policy to increase output from its full capacity level. (2+2+6)

लुकास का आपूर्ति संबंध किसी भी अर्थव्यवस्था में उत्पादन की संभावना को अपनी पूर्ण क्षमता स्तर से विचलित करने की संभावना का वर्णन कैसे करता है? नीति प्रभावी प्रस्ताव को सिद्ध करने के लिए तर्कसंगत अपेक्षा की क्या विशेषताएं हैं। सार्जेंट और वालेस (1976) द्वारा प्रस्तावित मॉडल का उपयोग करते हुए, अपनी पूर्ण क्षमता स्तर से उत्पादन बढ़ाने के लिए विस्तारवादी मौद्रिक नीति की अप्रभावीता की संभावना की व्याख्या कीजिए।

12. Explain the short-run variability and long-run constancy of the ratio of consumer expenditure to income with the level of income, in light of the *Permanent Income Hypothesis* of consumption postulated by Friedman. (10)

फ्रीडमैन द्वारा प्रतिपादित उपभोग की स्थायी आय परिकल्पना के आलोक में आय के स्तर के साथ उपभोक्ता व्यय और आय के अनुपात की अल्पकालिक परिवर्तनशीलता और दीर्घकालिक स्थिरता को स्पष्ट कीजिए।

13. Suppose a house in X-Y city can be rented for Rs. 30,000 monthly. The house depreciates at the rate of 5 percent per year, the annual interest is 10 percent, and mortgage interest is tax deductible, where the relevant tax rate is 30 percent. Following the arbitrage argument in residential investment,

(a) Determine the change in the house-price, in case of a reduction in the downpayment rate from 25 to 15 percent, given the expected annual growth rate of the house-price is at 8 percent. What do you infer regarding the cause of the housing price bubble from this change in house-price. (4+1)

(b) Determine the change in the house-price, in case of an increase in the expected annual growth rate of the house-price from 8 to 10 percent, given downpayment rate is at 25 percent. What do you infer regarding the cause of housing price bubble from this change in house-price. (4+1)

मान लीजिए कि X-Y शहर में एक घर 30,000 रुपये मासिक में किराए पर लिया जा सकता है। घर का प्रति वर्ष 5 प्रतिशत की दर से मूल्यह्रास होता है, वार्षिक ब्याज 10 प्रतिशत है, और बंधक ब्याज कर कटौती योग्य है, जहां प्रासंगिक कर दर 30 प्रतिशत है। आवासीय निवेश में आर्बिट्राज तर्क के बाद,

(क) नकद भुगतान दर में 25 से 15 प्रतिशत की कमी के मामले में, मकान की कीमत में परिवर्तन का निर्धारण कीजिए, यह देखते हुए कि मकान की कीमत की अपेक्षित वार्षिक वृद्धि दर 8 प्रतिशत है। मकान की कीमत में इस परिवर्तन से आवास मूल्य में उफान के कारण के बारे में आप क्या अनुमान लगाते हैं।

(ख) मकान की कीमत की अपेक्षित वार्षिक वृद्धि दर में 8 से 10 प्रतिशत की वृद्धि के मामले में, मकान की कीमत में परिवर्तन का निर्धारण कीजिए, क्योंकि नकद भुगतान दर 25 प्रतिशत है। मकान की कीमत में इस बदलाव से आवास मूल्य में उफान के कारण के बारे में आप क्या अनुमान लगाते हैं।

[This question paper contains 12 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 2309 **G**

Unique Paper Code : 2274001003

Name of the Paper : Principles of Microeconomics I

Name of the Course : **Generic Elective – Economics**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **five** questions.
3. Answers may be written either in English or Hindi; but the same medium should be used throughout the paper.

छात्रों के लिए निर्देश

1. इस प्रश्न-पत्र के मिलते ही ऊपर दिए गए निर्धारित स्थान पर अपना अनुक्रमांक लिखिए।

P.T.O.

2. किन्हीं पाँच प्रश्नों के उत्तर दीजिए ।

3. इस प्रश्न-पत्र का उत्तर अंग्रेजी या हिंदी किसी एक भाषा में दीजिए, लेकिन सभी उत्तरों का माध्यम एक ही होना चाहिए ।

1. Answer the following questions :

(a) Explain the concepts of producer surplus and consumer surplus. Consider a market for apples. Suppose the supply of apples rises. What happens to consumer and producer surplus at the new equilibrium in the market for apples? (9)

(b) What is Price Ceiling? Does a price ceiling fixed by the government always change the market outcome? Give reasons for your answer. (9)

निम्नलिखित प्रश्नों के उत्तर दीजिए :

(क) उत्पादक अधिशेष और उपभोक्ता अधिशेष की अवधारणाओं को समझाइये। सेब के लिए एक बाजार पर विचार करें। मान लीजिए सेब की आपूर्ति बढ़ जाती है। सेब के बाजार में नए संतुलन पर उपभोक्ता और उत्पादक अधिशेष का क्या होता है?

(ख) मूल्य सीमा क्या है? क्या सरकार द्वारा निर्धारित मूल्य सीमा हमेशा बाजार के नतीजे बदलती है? अपने उत्तर के कारण बताएं।

2. Answer the following questions :

(a) Rahul is consuming two goods X and Y. His income is Rs. 100. Price of Good X is Rs. 10 and the price of Good Y is Rs. 10. (9)

(i) Given the above information, draw the budget line for Rahul.

(ii) Suppose the price of Good X decreases to Rs. 5. How will the budget line for Rahul change?

(iii) Instead of a decrease in the price of Good X, Rahul's income increases to Rs. 200. How will the budget line for Rahul change, in comparison to Part (i)?

(b) What is an indifference curve? Explain the properties of indifference curves. (9)

निम्नलिखित प्रश्नों के उत्तर दीजिए :

(क) राहुल दो वस्तुओं X और Y का उपभोग कर रहा है। उसकी आय 100 रुपये है। वस्तु X की कीमत 10 रुपये है और वस्तु Y की कीमत 10 रुपये है।

(i) उपरोक्त जानकारी को देखते हुए राहुल के लिए बजट रेखा बनाएं।

(ii) मान लीजिए कि वस्तु X की कीमत घटकर 5 रुपये हो जाती है। राहुल के लिए बजट रेखा कैसे बदलेगी?

(iii) वस्तु X की कीमत में कमी के बजाय, राहुल की आय 200 रुपये तक बढ़ जाती है। भाग (i) की तुलना में, राहुल के लिए बजट रेखा कैसे बदलेगी?

(ख) उदासीनता वक्र क्या है? उदासीनता वक्रों के गुणों को समझाइये?

3. Answer the following questions :

(a) Whenever interest rates on savings increase, households tend to save more. Is this statement true or false? Explain with the help of income and substitution effects. (9)

(b) It is observed that at the existing equilibrium in the market, demand is highly inelastic while the supply is highly elastic. If the buyers have to pay a tax of Rs. T for each unit they buy, who will bear the higher burden of tax: buyer or seller? Explain why. (9)

निम्नलिखित प्रश्नों के उत्तर दीजिए :

(क) जब भी बचत पर ब्याज दरें बढ़ती हैं, तो परिवार अधिक बचत करने लगते हैं। यह कथन सही है या गलत? आय एवं प्रतिस्थापन प्रभावों की सहायता से समझाइये?

(ख) यह देखा गया है कि बाजार में मौजूदा संतुलन में, मांग अत्यधिक लोचदार है जबकि आपूर्ति अत्यधिक लोचदार है। यदि खरीदारों को प्रत्येक इकाई खरीदने पर T रुपये का कर देना होगा, तो कर का अधिक बोझ कौन वहन करेगा : खरीददार या विक्रेता? समझाइए क्यों?

4. Answer the following questions :

(a) What is a Production Possibility Curve (PPC)? Explain why under the usual assumptions, PPC is bowed outward (concave to the origin). What will be the shape of the PPC if the opportunity cost of Good Y in terms of Good X is constant? (9)

(b) What will be the shape of indifference curve if the two goods are : (9)

(i) Perfect Substitutes

(ii) Perfect Complements

(iii) One good is economic good, other is economic bad

निम्नलिखित प्रश्नों के उत्तर दीजिए :

(क) उत्पादन संभावना वक्र (पीपीसी) क्या है? बताएं कि सामान्य धारणाओं के तहत, पीपीसी बाहर की ओर (मूल बिंदु की ओर अवतल) क्यों झुका हुआ है। यदि वस्तु Y की अवसर लागत वस्तु X के संदर्भ में स्थिर है तो PPC का आकार क्या होगा?

(ख) यदि दो वस्तुएं हैं तो उदासीनता वक्रों का क्या होगा?

(i) पूर्ण प्रतिस्थापन

(ii) पूर्ण पूरक

(iii) एक वस्तु आर्थिक रूप से अच्छी है, दूसरी आर्थिक रूप से खराब

5. Answer the following questions :

(a) Differentiate between economies of scale and economies of scope. (9)

(b) Illustrate and explain the concept of Law of Variable Proportions. In this context, differentiate between increasing returns to a factor, constant returns to a factor and decreasing returns to a factor? (9)

निम्नलिखित प्रश्नों के उत्तर दीजिए :

(क) पैमाने की मितव्ययता और दायरे की मितव्ययता के बीच अंतर बताएं?

(ख) परिवर्तनीय अनुपात के नियम की अवधारणा को स्पष्ट करें और समझाएं। इस संदर्भ में, किसी कारक पर बढ़ते रिटर्न्, किसी कारक पर स्थिर रिटर्न और किसी कारक पर घटते रिटर्न के बीच अंतर करें?

6. Answer the following questions :

(a) Differentiate between explicit cost and implicit cost of a firm with the help of an example. In this context, explain the difference between economic profits and accounting profits. (9)

(b) Define perfect competition. What are its main characteristics? (9)

निम्नलिखित प्रश्नों के उत्तर दीजिए :

(क) एक उदाहरण की सहायता से किसी फर्म की स्पष्ट लागत और अंतर्निहित लागत के बीच अंतर करें। इस संदर्भ में, आर्थिक लाभ और लेखांकन लाभ के बीच अंतर स्पष्ट करें।

(ख) पूर्ण प्रतियोगिता को परिभाषित करें। इसकी मुख्य विशेषताएं क्या है?

7. Answer the following questions :

(a) Differentiate between shut down point and exit point of a firm in a perfectly competitive market. (9)

(b) Consider a firm in a perfectly competitive market for sugar. The market price for sugar is Rs. 24 per kg. The marginal cost function for this firm is given by: $MC = 3 + 7Q$. Calculate the quantity of sugar that this firm will be selling in the market at

equilibrium. What will be the total revenue, marginal revenue and average revenue for the firm? (9)

निम्नलिखित प्रश्नों के उत्तर दीजिए :

(क) पूर्ण प्रतिस्पर्धी बाजार में किसी फर्म के शट डाउन बिंदु और निकास बिंदु के बीच अंतर करें।

(ख) चीनी के लिए पूर्ण प्रतिस्पर्धी बाजार में एक फर्म पर विचार करें। चीनी का बाजार मूल्य 24 रुपये प्रति किलोग्राम है। इस फर्म के लिए सीमांत लागत फलन इस प्रकार दिया गया है : $MC = 3 + 7Q$ चीनी की मात्रा की गणना करें जो यह फर्म संतुलन पर बाजार में बेचेगी। फर्म का कुल राजस्व, सीमांत राजस्व और औसत राजस्व क्या होगा।

8. Answer the following questions :

(a) Consider a firm producing output (Y) using labour (L) and capital (K) in a perfectly competitive market. In each of the following production functions, determine whether the firm is facing

increasing returns to scale or decreasing returns to scale or constant returns to scale.

(i) $Y = L.K$

(ii) $Y = \sqrt{LK}$ (9)

(b) In continuation of Part (a), if suppose the firm is producing 100 units of output. Write the equation of isoquant for the two production functions given in Part (a). (9)

निम्नलिखित सवालों के जवाब दीजिए :

(क) पूर्ण प्रतिस्पर्धी बाजार में श्रम (L) और पूंजी (K) का उपयोग करके उत्पाद (Y) का उत्पादन करने वाली एक फर्म पर विचार करें। निम्नलिखित उत्पादन फलन में से प्रत्येक में, यह निर्धारित करें कि क्या फर्म को पैमाने पर बढ़ते रिटर्न या पैमाने पर घटते रिटर्न या पैमाने पर निरंतर रिटर्न का सामना करना पड़ रहा है।

(i) $Y = L.K$

(ii) $Y = \sqrt{LK}$

(ख) भाग (a) की निरंतरता में, यदि मान लें कि फर्म 100 इकाइयों का उत्पादन कर रही है। भाग (a) में दिए गए दो उत्पादन कार्यों के लिए समुत्पाद का समीकरण लिखें।